7-2.3 Mode-Matching Analysis

The analysis above fails to give the input match and is unable to handle discontinuities along the horn cone, such as the slots of the corrugated horn. Mode-matching analysis solves these problems. For the simple conical horn we approximate the cone as a series of short-length cylindrical sections joined with small steps. We expand the field in each cylindrical section as a series of TE and TM modes that propagate from the input waveguide to the aperture. At the waveguide input only the lowest order mode propagates, but we must include the evanescent modes because the waveguides are short and significant power is carried by them. The aperture size determines required the number of modes at 12 per wavelength for accurate results [aa]. The discontinuities between the cylindrical excite higher order modes required to match the fields along these steps and the lower order modes couple into higher ones. The analysis uses the number of modes required by the aperture size to determine the matrix size. Every mode leads to a cascaded transmission line analysis through the horn with coupling through the modes caused by the discontinuities. We start with a single input mode that expands into the many output modes.

The modal solution of a waveguide produces a set of orthogonal functions where an integral over the waveguide cross-section of the product of two of these functions times some weighting function gives zero for unequal modes. This integral scalar product is not unique but can be any integral that produces a finite value for coupling of the mode into itself and zero for unequal modes. One such integral is the reaction theorem Eq. (2-35) used for coupling. James [bb] used a variation of the reactance theorem to compute the s-parameter matrices where the integrals are taken over the waveguide discontinuity cross sections. Because the sections have different radii, scalar products (integrals) between unmatched modes are no longer zero and the modes couple. Circular waveguides fed by the TE_{11} mode only use TE_{1m} and TM_{1m} modes because the steps do not excite different φ modes from the input. This characteristic does not hold for rectangular waveguide modes because all modes intermix at steps. However, the sectorial rectangular horns have a constant mode index either for the *x*-axis or *y*-axis depending on the non-flared coordinate. Although many steps and modes are required for accurate results, the costly matrix inversion of a MoM solution or the many cells of either a finite element or time domain method are not needed. In comparison the method is very fast.

The mode-matching analysis uses the aperture fields to compute the far-field patterns. Adding currents on the exterior of the horn found by a MoM solution using the aperture fields for excitation improves the analysis. A body of revolution (BOR) MoM analysis is applied to the circular horns which add little run time to the analysis. Two commercial codes for mode-matching are CHAMP (TICRA) and the Mician *Microwave Wizard*. Each of these codes includes optimization techniques so that they can serve a design tools. The dimensions only need to be close and geometry variables defined in the program are manipulated to achieve various goals.

CHAMP was applied to the two 22-dB circular horns designed above for 8 GHz operation. The geometry starts with an input waveguide with a suggested radius $a_i = 3\lambda_c/(2\pi)$, 17.892 mm. The CHAMP analysis predicted 22.06 dB gain for 22-dB gain horn with S = 0.2. Adding the exterior and including the BOR MoM analysis altered the result to 22.03 dB gain; the exterior has little effect. For the optimum horn with S = 0.39 CHAMP predicted a gain of 21.98 dB, but when the exterior currents were added it dropped to 21.87 dB. To test the design technique used above horns were designed with 15 dB and 13 dB gain. The CHAMP analysis predicted 14.94 dB and 13.07 dB, respectively with added exterior currents only changing the gain to 13.11 dB.