7-3.2 Corrugation Design

The corrugations present a capacitive reactance to the passing wave. When a corrugated surface is inductive, it will support surface waves. The depth of corrugations must be between λ/4 and λ/2. Less than λ/4 or greater than λ/2, it is inductive. Between 3λ/2 and λ it will be capacitive again, but this second passband is seldom used. A quarter-wavelength corrugation depth balances the two modes and gives the best results. The corrugations need be only λ /4 at the aperture. Before the aperture we find it better to deepen the slots. Quarter-wavelength deep corrugations mismatch the horn in the transition region, where the TM_{11} mode is generated from the TE_{11} mode and depths approaching λ /2 have the least effect on match.

To design for a particular band, limited to about 1.5:1 for a good match, we design with tapered corrugation depths. We make the corrugations λ /4 deep at the aperture at the low-frequency end. The high-frequency band edge determines the corrugation depth at just short of λ /2 in the throat. The horn needs at least five corrugations per wavelength, pitch *p*, along the slant radius while broadband designs may need ten per wavelength at the center frequency. The pitch-to-width ratio, *δ*, range is 0.7 to 0.9. The first few corrugations can be used to match the horn to the waveguide, and we can improve the match by shaping the corrugation widths [14]. The slots should be as wide as practical spacers will allow. Mechanical considerations, such as shock and vibration, will determine the limits on the thinness of spacers, but corrugations greatly increase the strength of the bell.

The circular geometry of the horn changes the corrugation depth necessary for the balanced HE_{11} mode from λ /4. An empirical formula for the depth is given by [17a]

$$d = \frac{\lambda_c}{4} \exp \frac{1}{2.114(k_c a)^{1.134}} \qquad k_c a > 2 \tag{7-23}$$

The value $k_c = 2\pi / \lambda_c$ depends on the desired bandwidth. A narrowband design, $f_{\max} \le 1.14 f_{\min}$, uses the geometric mean for center frequency $f_c = \sqrt{f_{\min} f_{\max}}$ while a broadband design, $1.4 f_{\min} \le f_{\max} \le 2.4 f_{\min}$ uses a center frequency $f_c \approx 1.2 f_{\min}$. A suggested input waveguide radius is $a_i = 3\lambda_c / (2\pi)$ [17a].
We increase the depth slightly at the horn aperture.

We use a linearly tapered section at the feed waveguide input as a mode converter that contains (N_{MC}) 5 to 7 slots to convert the input TE_{11} mode to HE_{11}.

$$d_j = \left[\sigma - \frac{j-1}{N_{MC}} \left(\sigma - 0.25 \exp \frac{1}{2.114(k_c a_j)^{1.134}} \right) \right] \lambda_c$$

The parameter σ ranges from 0.4 to 0.5 with a suggested value of 0.42 (depth of 1st corrugation) in wavelengths. Equation (7-23) is used for all corrugations in the bell region where the corrugation radius is located at the outer tooth and the depth is along the inner edge of the tooth.

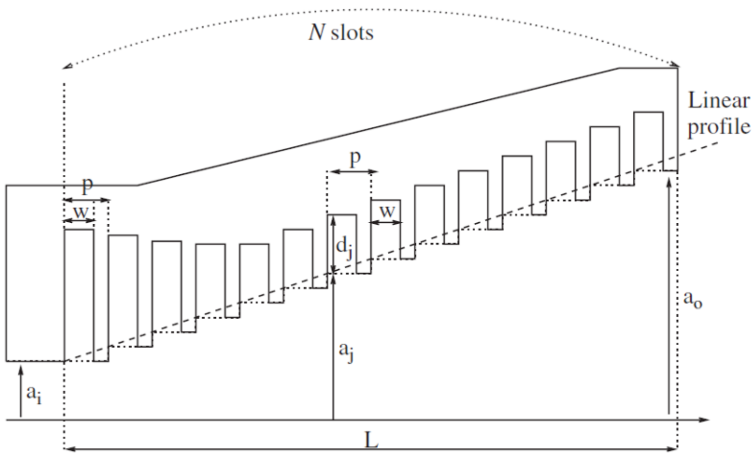

Figure 7-aa. Corrugation radius and depth in linear profile horn (*Modern Antenna Handbook*, Wiley, p. 102)

Another possibility for the mode converter is to use ring-loaded-slots for wideband horns. Reference [17a] gives the geometry and design equations for these. Variable pitch-to-width mode converters are also be used for wideband designs.

The figure above shows a linear profile along the bell. Other types of profiles produce either shorter horns or less impedance discontinuities. Given the position along the horn *z*, the radius *a(z)* becomes a function of the total length, *L*. The equations below use the parameter ρ with the range 0.5 to 5 with the most used value of two.

Profile	**Formulation**
Linear	$a(z) = a_i + (a_o - a_i)z/L$
Sinusoidal	$a(z) = a_i + (a_o - a_i)\left[(1-A)z/L + A\sin^{\rho}(\pi z/(2L))\right]$ where $A \in [0;1]$
Asymmetric Sine-squared	$a(z) = a_i + \dfrac{2(a_o - a_i)}{1+\gamma}\sin^2\left(\dfrac{\pi z}{4L_1}\right)$ for $0 \le z \le L_1$ $a(z) = a_i + \dfrac{2(a_o - a_i)}{1+\gamma}\left\{\gamma\sin^2\left[\dfrac{\pi\left(z + L_2 - L_1\right)}{4L_2}\right] + \dfrac{1-\gamma}{2}\right\}$ for $L_1 \le z \le L$, $L = L_1 + L_2$, and $\gamma = L_2/L_1$
Tangential	$a(z) = a_i + \left(a_o - a_i\right)\left[\left(1-A\right)\dfrac{z}{L} + A\tan^{\rho}\left(\dfrac{\pi z}{4L}\right)\right]$ where $A \in [0;1]$
Simple Polynomial	$a(z) = a_i + \left(a_o - a_i\right)\left[\left(1-A\right)\dfrac{z}{L} + A\left(\dfrac{z}{L}\right)^{\rho}\right]$ where $A \in [0;1]$
Exponential	$a(z) = a_i \exp\left[z\ln\left(a_o/a_i\right)/L\right]$
Hyperbolic	$a(z) = \sqrt{a_i^2 + z^2\left(a_o^2 - a_i^2\right)/L^2}$

Polynomial	$a(z)=a_i+(\rho+1)(a_o-a_i)\left[1-\frac{\rho z}{(\rho+1)L}\right]\left(\frac{z}{L}\right)^{\rho}$

17a. Christophe Granet and Graeme L. James, Design of Corrugated Horns: A Primer, *IEEE AP-S Magazine*, vol. 47, No. 2, April 2005, pp. 76-84.