8-20.5 Axisymmetric Dual Shaped Reflector Design

We use a system of 3 differential equations to define the surface of an axisymmetric dual reflector which we solve to produce a desired aperture distribution in a geometric optics sense. The final design pattern needs to be computed using a computational electromagnetic code using physical optics combined with PTD or BOR-MoM. The finite size of the reflectors and the blockage of the subreflector alter the final pattern by scattering and diffraction.

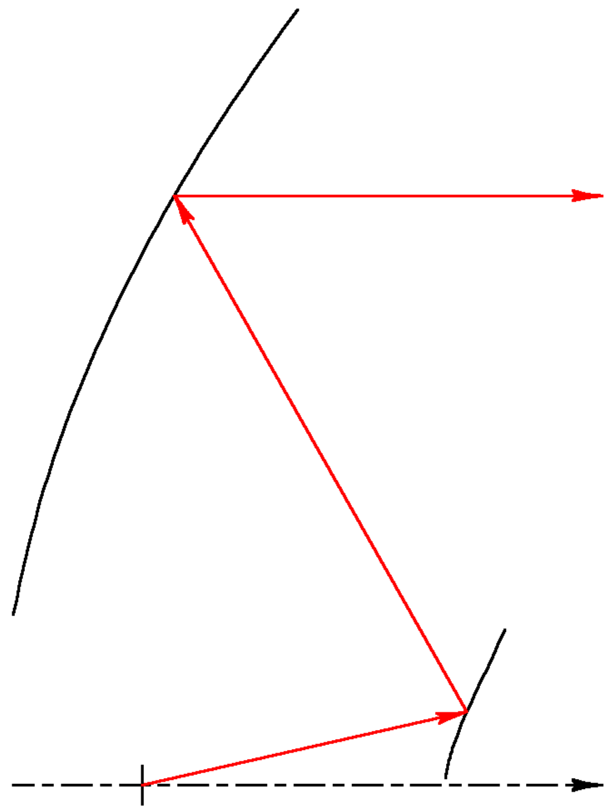

Figure 8-20.5.1 Cassegrain type dual shaped reflector

Figure 8-20.5.1 shows a dual reflector design converting a 46.4° 10 dB beamwidth feed into a uniform aperture distribution using geometric optics. The main reflector position is (r, z_m) and the subreflector position is (s, z_s) for the rays traced through the dual reflector as shown. We start with a feed angle θ which traced through the dual reflector reflects to a horizontal ray through an angle β. Three differential equations determine the ray trace.

$$\frac{d\theta}{dr} = \frac{rI(r)P_f}{F(\theta)\sin\theta} \qquad \frac{d\rho}{d\theta} = \rho\frac{d\theta}{dr}\tan\frac{\theta+\beta}{2} \qquad \frac{dz_m}{dr} = \tan(\beta/2)$$

I(r) is the power distribution of the desired aperture distribution and *F(θ)* is the power feed pattern at the feed angle θ. The reflector ranges from an inner radius $r_{\min}$ to $r_{\max}$ which means the subreflector will not reflect rays into the central inner region similar to the displaced axis reflector. The first differential equation contains a power factor.

$$P_f = \frac{\int_0^{\theta_{\max}} F(\theta)\sin\theta d\theta}{\int_{r_{\min}}^{r_{\max}} rI(r)dr}$$

The radial distance from the feed to the subreflector is $\rho(\theta)$ and position $s = \rho(\theta)\sin\theta$ and $z_s = \rho(\theta)\cos\theta$ for an origin at the phase center of the feed. The ray from the feed to maximum edge of subreflector reflects to the out rim of the main reflector. Notice that the first differential equation cannot start where $\theta = 0$ because it will be infinite. To avoid this problem we start at the outer rim of the main reflector and decrement r and numerically solve the system of differential equations.

The circular aperture distributions of Chapter 4 range from 0 to 1 so we will solve a normalized main reflector (r_{max} = 1). In this normalized dual reflector the distance of the main reflector rim to the feed is A and the distance to the rim of the subreflector is B. The initial main reflector angle β_i

$$\beta_i = \tan^{-1}\frac{r_{max} - s_{max}}{A+B} \quad \text{and general } \beta = \tan^{-1}\frac{r-s}{z_s - z_m}$$

A Runge-Kutta solution of the three differential equation steps from $r = 1$ to the inner radius.

The design of the dual reflector Figure 8-20.5.1 uses $A = -0.2$, that is, the feed is located behind the main reflector rim. The subreflector rim is located $B = 0.467$ from the feed with a maximum radius of 0.2 which producing a maximum half feed subtended angle of 23.18°. The main reflector has a central hole of normalized radius 0.2 which matches the subreflector. The program DUSREF iterates the differential equations for this Cassegrain type shaped dual reflector with output file DUSREFC.OUT. The aperture distribution options are: uniform, circular Gaussian, and Taylor for given sidelobes.

The Gregorian reflector Figure 8-20.5.2 can be solved using the same differential equations.

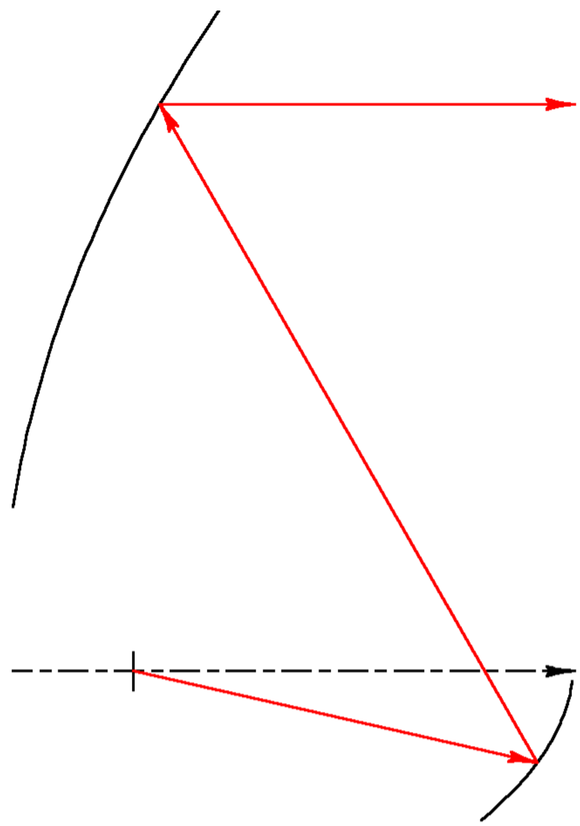

Figure 8-20.5.2 Gregorian type dual shaped reflector

This solution starts with a negative feed angle and the first differential equation increases the feed to subreflector distance $\rho(\theta)$ because $\sin\theta$ is negative as well as the decrement of the main reflector *r*. The subreflector radius *s* is negative throughout the increment of the system of DE's. Figures 8-20.5.1 and 8-20.5.2 have the same feed to subreflector rim and feed to main reflector rim spacing, as well as, subreflector radius.

Displaced Axis Shaped Dual Reflector

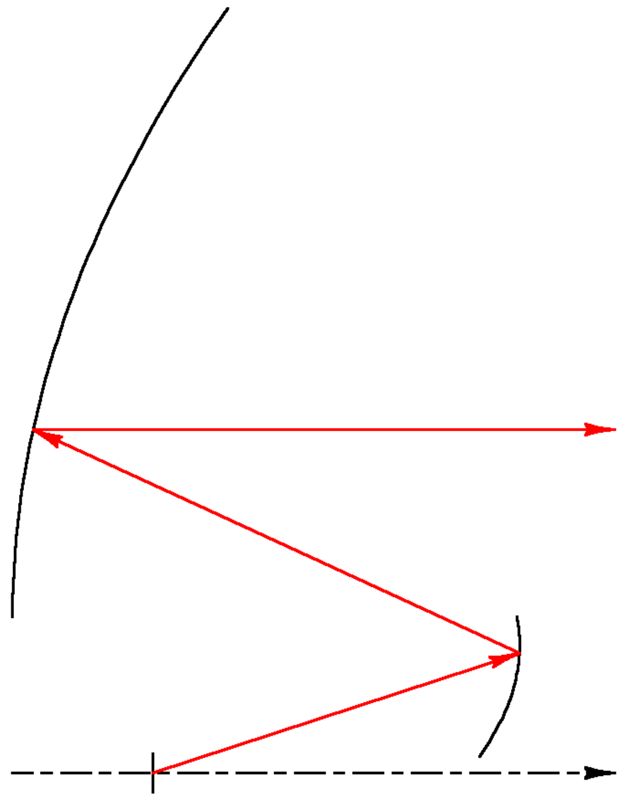

Figure 8-20.5.3 Displaced Axis Dual Shaped Reflector

The displaced axis reflector has a crisscross feeding where the ray which strikes the subreflector outer rim feeds the edge of the center hole. The ray from center of the subreflector reflects to the main reflector rim. The normal displaced axis reflector Section 8-16 has zero amplitude at the aperture edge Figure 8-17. The iteration of the differential equations system uses the main reflector aperture *r* which starts at the reflector hole and moves outward. The initial subreflector point is the rim and moves toward the center as the differential equations are solved. The differential equations are almost the same except for a sign change.

$$\frac{d\theta}{dr} = \frac{-rI(r)P_f}{F(\theta)\sin\theta} \qquad \frac{d\rho}{d\theta} = \rho\frac{d\theta}{dr}\tan\frac{\theta+\beta}{2} \qquad \frac{dz_m}{dr} = \tan(\beta/2)$$

For reflector *B* is the axis distance from the feed to the subreflector rim and *A* is the axial distance from the main reflector hole to the feed. Using these initial constants we compute the initial β, $\rho(\theta)$, and subreflector position $s = \rho(\theta)\sin\theta$ and $z_s = \rho(\theta)\cos\theta$ while incrementing the system of differential equations.

$$\beta_i = \tan^{-1}\frac{r_{\min} - s_{\max}}{A+B} \quad \text{and general } \beta = \tan^{-1}\frac{r-s}{z_s - z_m}$$

Aperture distributions with a finite edge value produce a design with a hole in the subreflector. Of course, the hole can be filled which produces main reflector spillover. The program DUSADE generates the design.

A series of shaped dual reflectors were designed, sized to 100λ, and analyzed using both GRASP and CHAMP. Table 1 lists a summary of designs and their characteristics.

Table 1 Dual Axisymmetric Shaped Reflector with 100λ Main Reflector Comparison

Input File	Type	Subrefl. Diameter	Main Refl.Hole	Feed Taper	GRASP Gain, dB	CHAMP Gain, dB	Sidelobe dB	Eff. (%)
Dusrefc2	Cassegrain	20λ	20λ	-10 dB	48.83	48.86	14.36	77.4
Dusrefc1	Cassegrain	20	0	-10	48.76	48.82	14.92	76.0
Dusrefc6	Cassegrain	20	0	-15	49.12	49.06	14.69	82.6
Dusrefc3	Cassegrain	10	10	-10	48.80	48.76	15.72	76.8
Dusrefc5	Cassegrain	10	0	-15	49.09	48.92	14.68	82.1
Dusrefc4	Cassegrain 30 Taylor	10	10	-10	48.24	48.19	19.34	67.5
Dusrefc7	Cassegrain -15 Gauss.	10	10	-10	47.97	48.00	23.12	63.4
Dusrefg2	Gregorian	20	20	-10	48.85	48.87	14.70	77.7
Dusrefg1	Gregorian	20	0	-10	48.79	48.79	15.31	76.5
Dusrefg3	Gregorian	10	10	-10	48.81	48.89	15.98	76.9
Dusrefg6	Gregorian	10	0	-10	48.83	48.80	16.41	77.2
Dusrefg5	Gregorian	20	0	-15	49.17	49.02	15.24	83.5
Dusrefg7	Gregorian	10	0	-15	49.20	49.11	16.69	84.1
Dusade2	ADE	20	20	-10	48.78	48.72	14.26	76.4
Dusade3	ADE	10	10	-10	48.90	48.70	16.50	78.6
Dusade6	ADE	20	20	-15	49.07	48.97	14.50	81.6
Dusade7	ADE	20	0	-15	49.04	49.03	15.30	81.1
Dusade5	ADE	10	10	-15	48.96	48.76	14.50	79.7

Consider three designs using a 100λ diameter main reflector and a 20λ diameter subreflector using a -10 dB feed taper in the direction the rim of the subreflector. The main reflector has a matching 20λ central hole.

Cassegrain Type Dual Reflector for Uniform Distribution DUSREF output

```
Normalized Shaped Axisymmetric Dual Reflector

Main Rim to Feed along axis:          -0.2000
Feed to Subreflector Rim along axis:   0.4670
Subreflector max. radius:              0.2000
Main reflector inner radius:           0.2000
Feed Half Subtended Angle:             23.184
Cassegrain geometry
Feed beamwidth:  46.40 level dB:  10.0
Uniform aperture distribution
Calculation Main Aperture interval:    0.002
Output Aperture Interval:    0.020
```

```
 Feed      Subreflector      Main Reflector
 Angle       Z        Radius      Z        Radius Path length

 23.18   0.46700   0.20000   0.20000   1.00000 1.15140E+00
 21.51   0.45814   0.18058   0.18563   0.98000 1.15140E+00
 20.19   0.45130   0.16592   0.17139   0.96000 1.15140E+00
 19.07   0.44571   0.15411   0.15733   0.94000 1.15140E+00
 18.10   0.44099   0.14418   0.14347   0.92000 1.15141E+00
 17.24   0.43690   0.13559   0.12984   0.90000 1.15141E+00
 .
 .
 .
  1.97   0.39144   0.01343  -0.16185   0.24001 1.15141E+00
  1.35   0.39071   0.00924  -0.16565   0.22001 1.15141E+00
  0.04   0.38919   0.00025  -0.16919   0.20001 1.15141E+00
```

The listing above illustrates the differential equation stepping along the main reflector unit radius aperture that starts at the outer rim and moves toward the central hole. The listing gives the feed angle that starts at the subreflector outer rim and progresses to almost zero. The last column lists the path length of rays traced from the feed through the two reflectors to the aperture plane and shows the expected constant value producing a uniform phase aperture.

Gregorian Type Dual Reflector for Uniform Distribution DUSREF output

```
Normalized Shaped Axisymmetric Dual Reflector

Main Rim to Feed along axis:           -0.2000
Feed to Subreflector Rim along axis:    0.4670
Subreflector max. radius:               0.2000
Main reflector inner radius:            0.2000
Feed Half Subtended Angle:              23.184
Gregorian geometry
Feed beamwidth:  46.60 level dB:  10.0
Uniform aperture distribution
Calculation Main Aperture interval:    0.002
Output Aperture Interval:    0.020

 Feed      Subreflector      Main Reflector
 Angle       Z        Radius      Z        Radius Path length

-23.18   0.46700  -0.20000   0.20000   1.00000 1.53737E+00
-21.53   0.47963  -0.18922   0.18420   0.98000 1.53737E+00
-20.21   0.48957  -0.18025   0.16884   0.96000 1.53737E+00
-19.10   0.49784  -0.17244   0.15389   0.94000 1.53737E+00
-18.14   0.50495  -0.16542   0.13933   0.92000 1.53737E+00
-17.28   0.51120  -0.15900   0.12515   0.90000 1.53737E+00
 .
 .
 .
 -1.97   0.58793  -0.02024  -0.15855   0.24001 1.53737E+00
 -1.36   0.58903  -0.01397  -0.16177   0.22001 1.53737E+00
 -0.04   0.59101  -0.00037  -0.16462   0.20001 1.53737E+00
```

The listing above illustrates the differential equation stepping along the main reflector unit radius aperture that starts at the outer rim and moves toward the central hole. The listing gives the feed angle that starts negative at the subreflector outer rim and progresses to almost zero since the Gregorian dual reflector has a caustic ray trace from the subreflector to the main. Notice that listing gives negative radii for the subreflector.

Displaced Axis (ADE) Type Dual Reflector for Uniform Distribution DUSADE output

```
 Normalized Shaped Axisymmetric Dual ADE Reflector

 Main Hole to Feed along axis:          0.1800
 Feed to Subreflector Rim along axis:   0.4670
 Subreflector max. radius:              0.2000
 Main reflector inner radius:           0.2000
 Feed Half Subtended Angle:             23.184
 Feed beamwidth:  46.60 level dB:  10.0
 Uniform aperture distribution
 Calculation Main Aperture interval:    0.002
 Output Aperture Interval:    0.020

  Feed      Subreflector      Main Reflector
  Angle      Z       Radius      Z       Radius Path length

  23.18   0.46700   0.20000  -0.18000   0.20000 1.33502E+00
  22.80   0.46767   0.19656  -0.17982   0.22000 1.33502E+00
  22.40   0.46828   0.19299  -0.17928   0.24000 1.33502E+00
  21.99   0.46883   0.18932  -0.17837   0.26000 1.33502E+00
  21.57   0.46931   0.18557  -0.17710   0.28000 1.33502E+00
  21.15   0.46971   0.18175  -0.17546   0.30000 1.33502E+00
  .
  .
  .
   4.22   0.42592   0.03145   0.08218   0.95999 1.33502E+00
   2.97   0.41953   0.02179   0.09632   0.97999 1.33502E+00
   0.11   0.40448   0.00074   0.11094   0.99999 1.33502E+00
```

The displaced axis reflector differential equation main reflector aperture radius increment starts at the inner main reflector aperture radius and increments to the outer rim. The outer subreflector rim feed ray reflects to the inner radius of the main reflector.

The programs generate rotational symmetric reflector (*.rsf) files of the main and subreflector for use in GRASP or CHAMP. CHAMP can further optimize the pattern when the reflectors are added as splines with variables into the geometry file (*.tor). Below is the listing of a main reflector addition created by the programs. The spline addition to CHAMP can be used to create the reflector surfaces for analysis. By expressing the *z*-axis positions as variables, they can be used as optimization variables to generate a desired pattern in CHAMP.

```
z_focal_point   real_variable
(
  value    : 0.0
)

zmain1   real_variable
(
   value    : -2.467600E-01
)
.
.
.
```

```
zmain9   real_variable
(
   value    :  3.000000E-01
)

main_reflector  spline_circ_sym_reflector
(
  z_offset          : "ref(z_focal_point)" m ,
  length_unit       : m ,
  nodes             : table
    (
     3.000000E-01   "ref(zmain1)"
     4.500000E-01   "ref(zmain2)"
     6.000000E-01   "ref(zmain3)"
     7.500000E-01   "ref(zmain4)"
     9.000000E-01   "ref(zmain5)"
     1.050000E+00   "ref(zmain6)"
     1.200000E+00   "ref(zmain7)"
     1.350000E+00   "ref(zmain8)"
     1.500000E+00   "ref(zmain9)"
    )
)
```

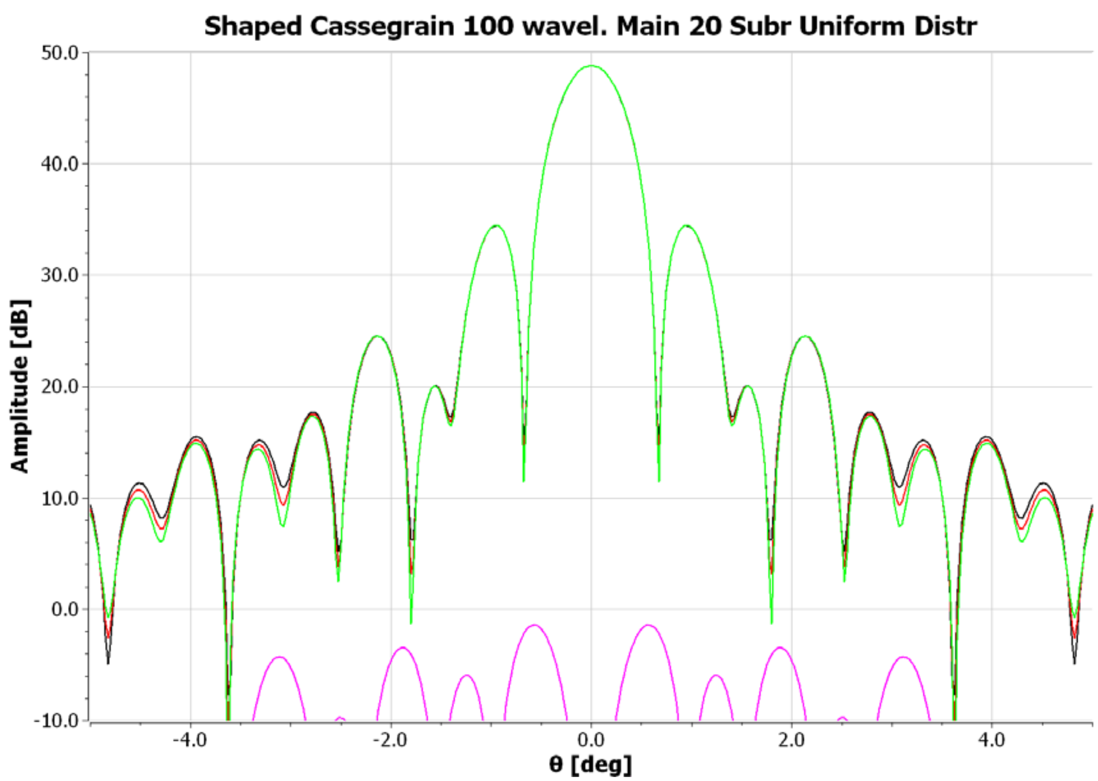

Figure 8-20.5.4 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 20λ subreflector and main hole using -10 dB Feed Taper - GRASP Analysis

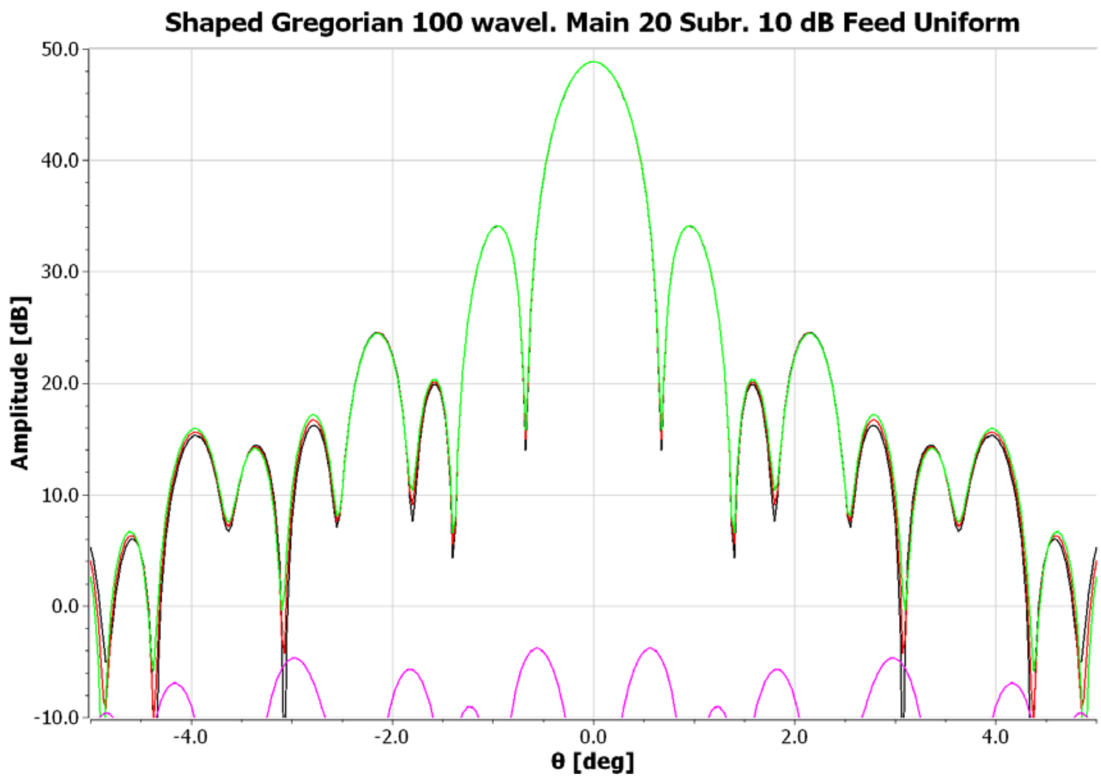

Figure 8-20.5.5 Main Beam Pattern of 100λ diameter Shaped Gregorian Reflector 20λ subreflector and main hole using -10 dB Feed Taper - GRASP Analysis

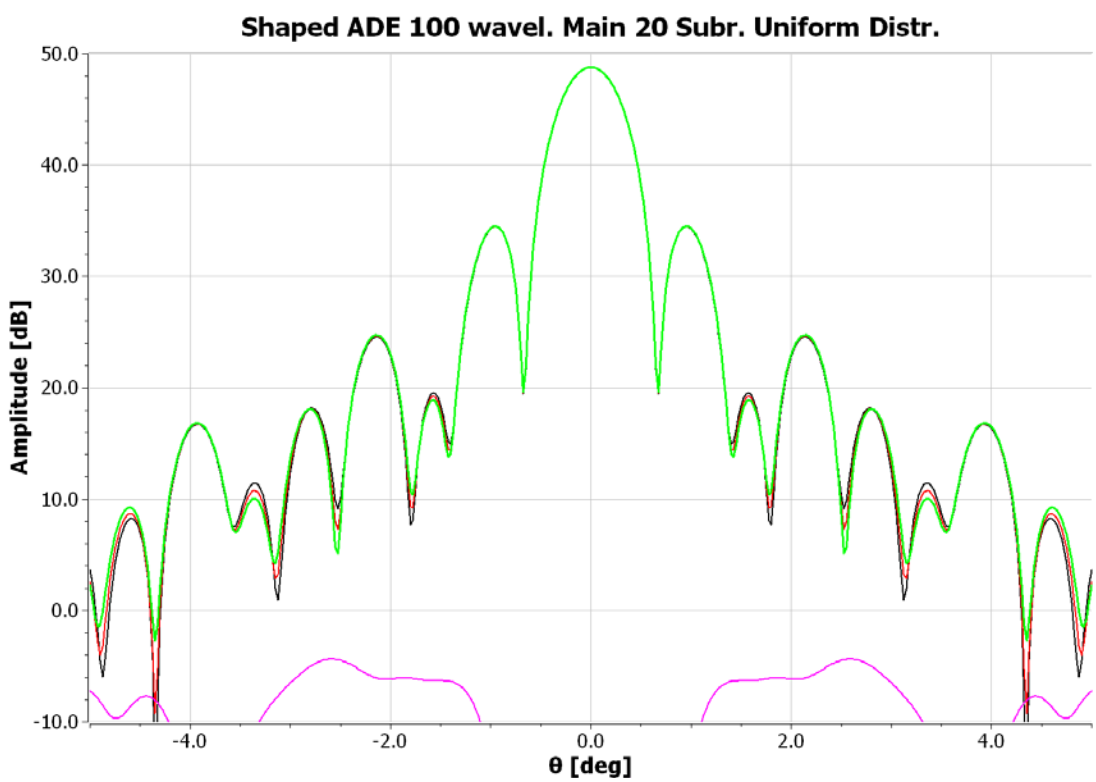

Figure 8-20.5.6 Main Beam Pattern of 100λ diameter Shaped Displaced Axis Reflector 20λ subreflector and main hole using -10 dB Feed Taper - GRASP Analysis

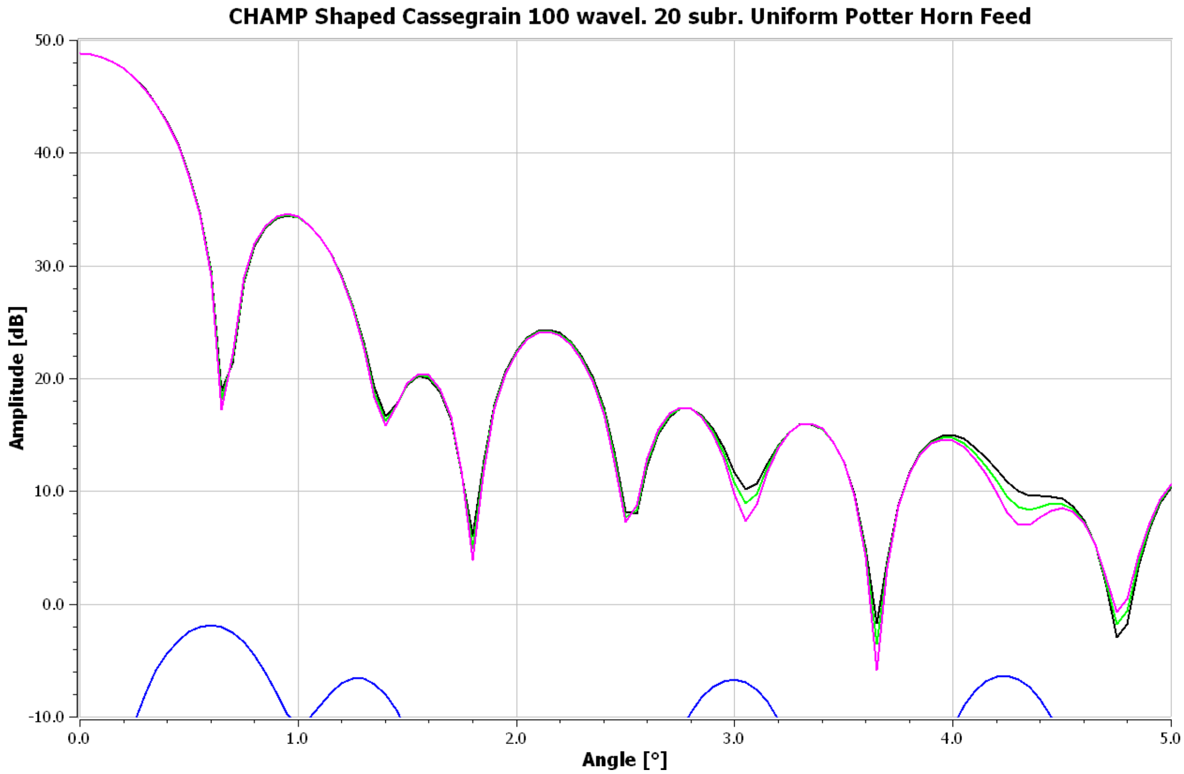

Figure 8-20.5.7 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 20λ subreflector and main hole using -10 dB Feed Taper - CHAMP Analysis using *.rsf inputs

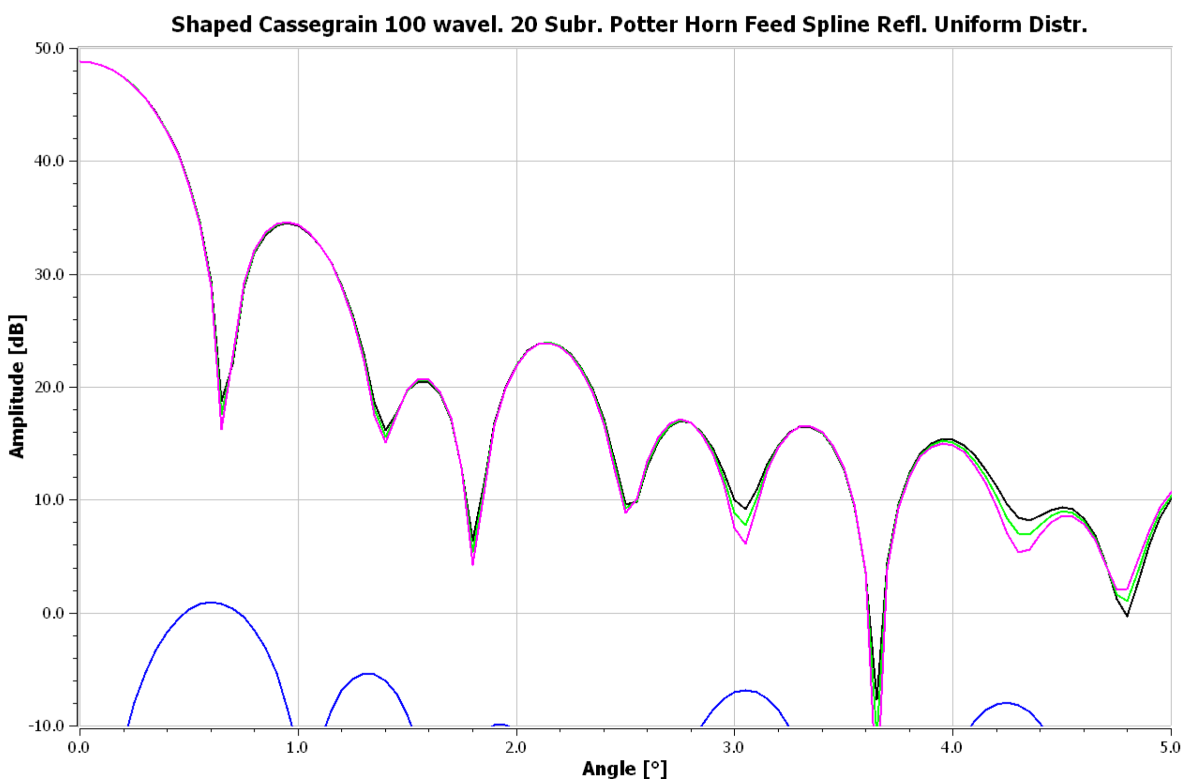

Figure 8-20.5.8 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 20λ subreflector and main hole using -10 dB Feed Taper - CHAMP Analysis using spline reflector inputs

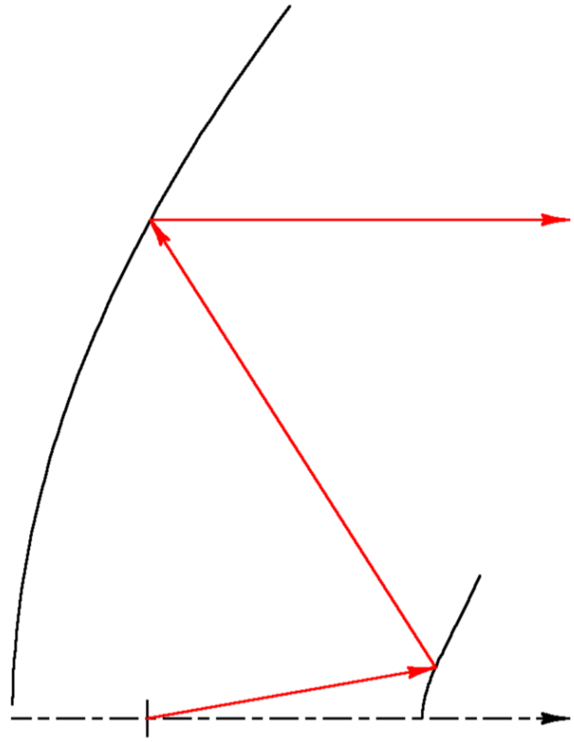

Figure 8-20.5.9 Cassegrain type dual shaped reflector w/o Central Hole

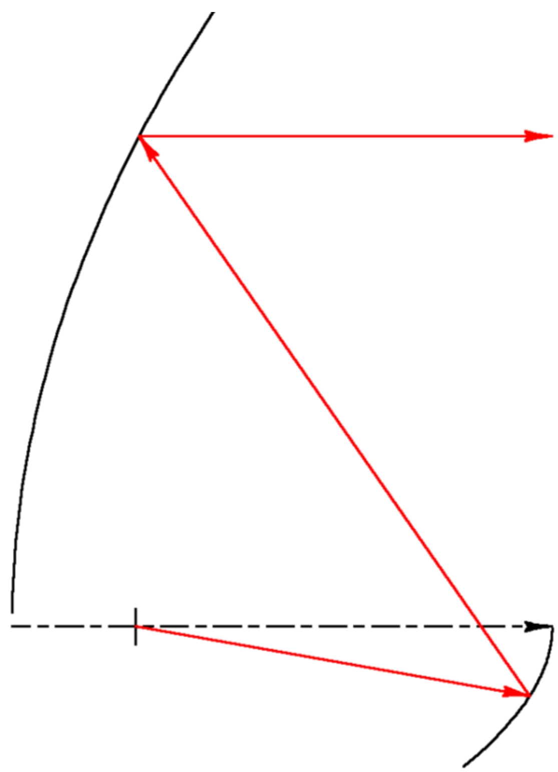

Figure 8-20.5.10 Gregorian type dual shaped reflector w/o Central Hole

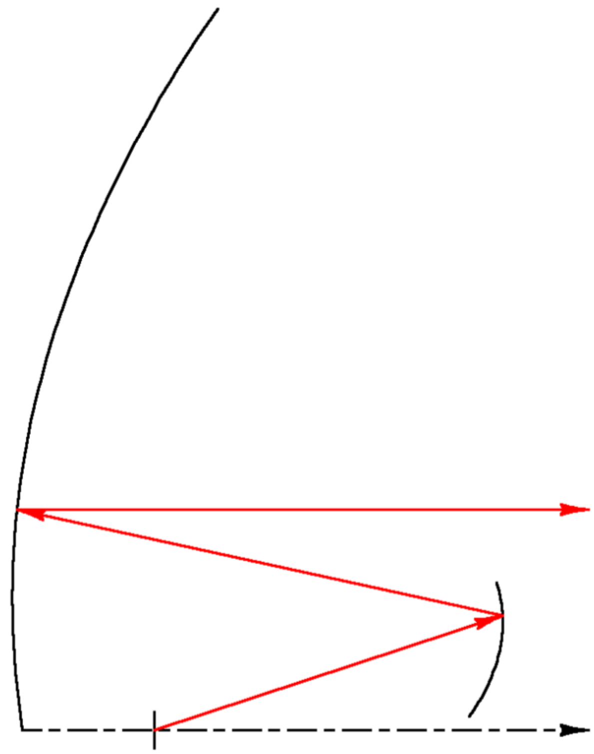

Figure 8-20.5.11 Displaced Axis (ADE) type dual shaped reflector w/o Central Hole

Figures 9, 10, and 11 illustrate three shaped dual reflector designs where the central hole has been eliminated. The feed subreflector edge taper is -15 dB. DUSADE is able to design a shaped ADE reflector without the usual main reflector central hole. Of course, in all cases the subreflector will block the central portion of main reflector radiation. However, filling the main reflector hole will reduce the backlobe.

These three antennas were analyzed using both GRASP and CHAMP. The results show nearly identical main beam patterns. The caustic reflection on a single side of the displaced axis reflector has no effect once the main- and sub-reflectors have been designed to reflect rays into the central main reflector hole. Figures 12, 13, and 14, given below, illustrate that the three reflectors produce nearly identical main beam patterns. Figure 15 plots the full pattern of the shaped dual Cassegrain type reflector. Closing the main reflector central hole decreases the backlobe (Figure 15).

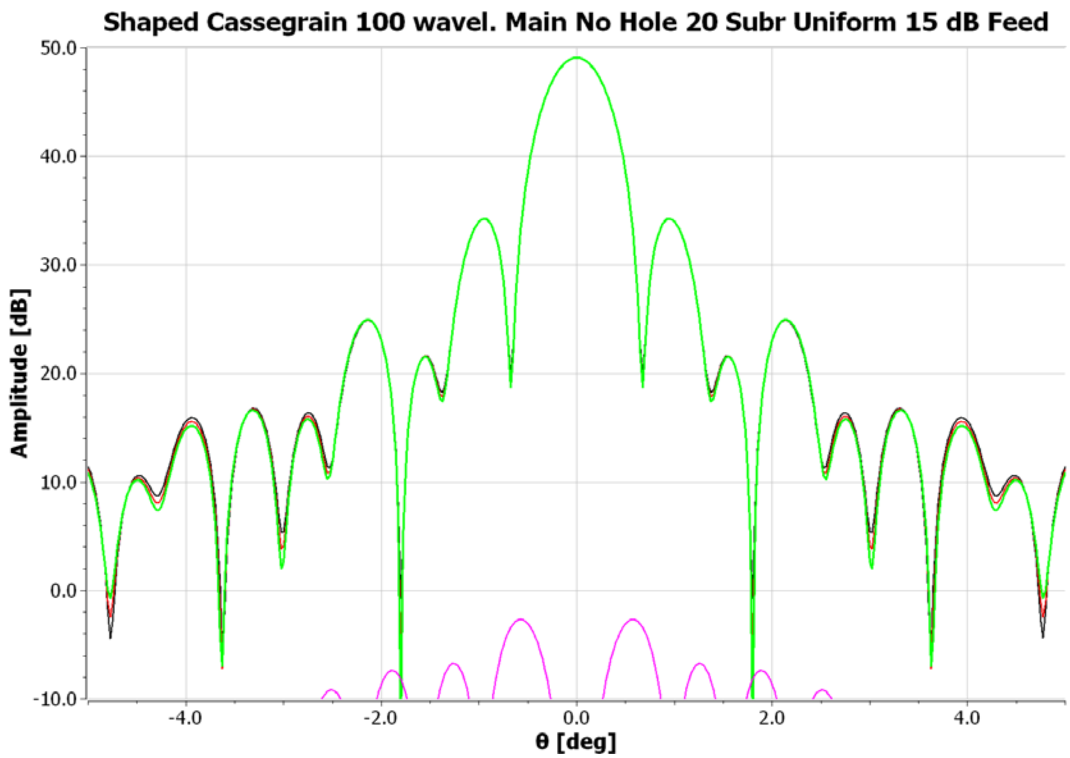

Figure 8-20.5.12 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - GRASP Analysis

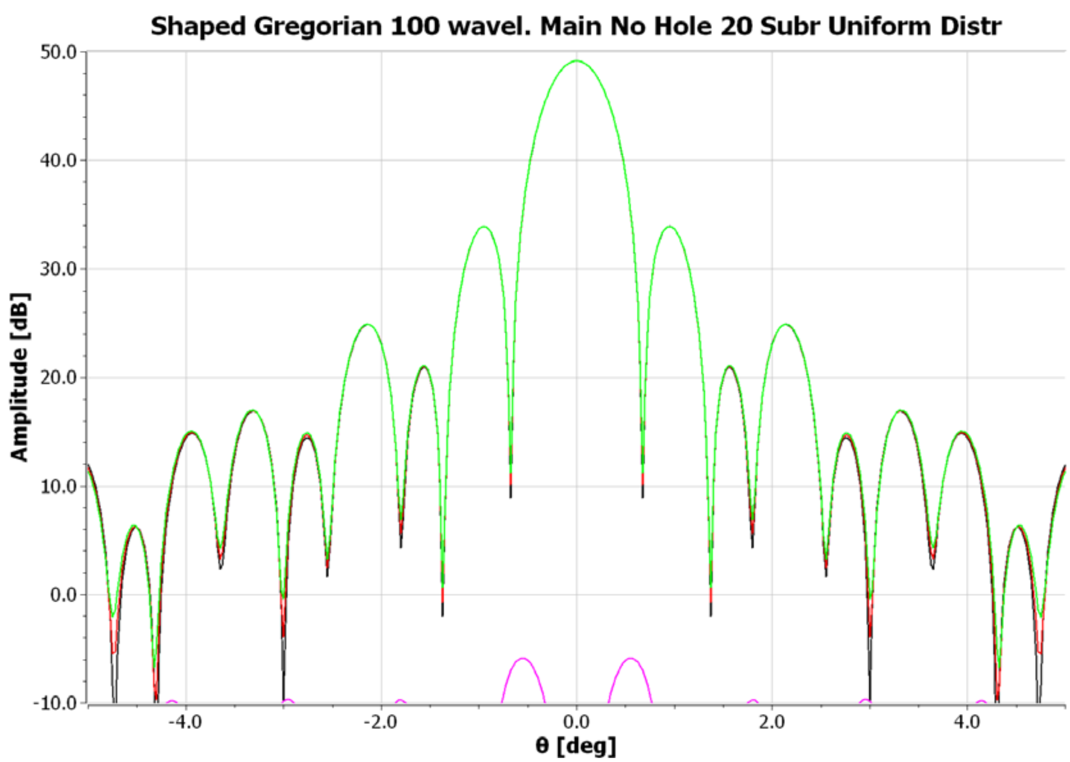

Figure 8-20.5.13 Main Beam Pattern of 100λ diameter Shaped Gregorian Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - GRASP Analysis

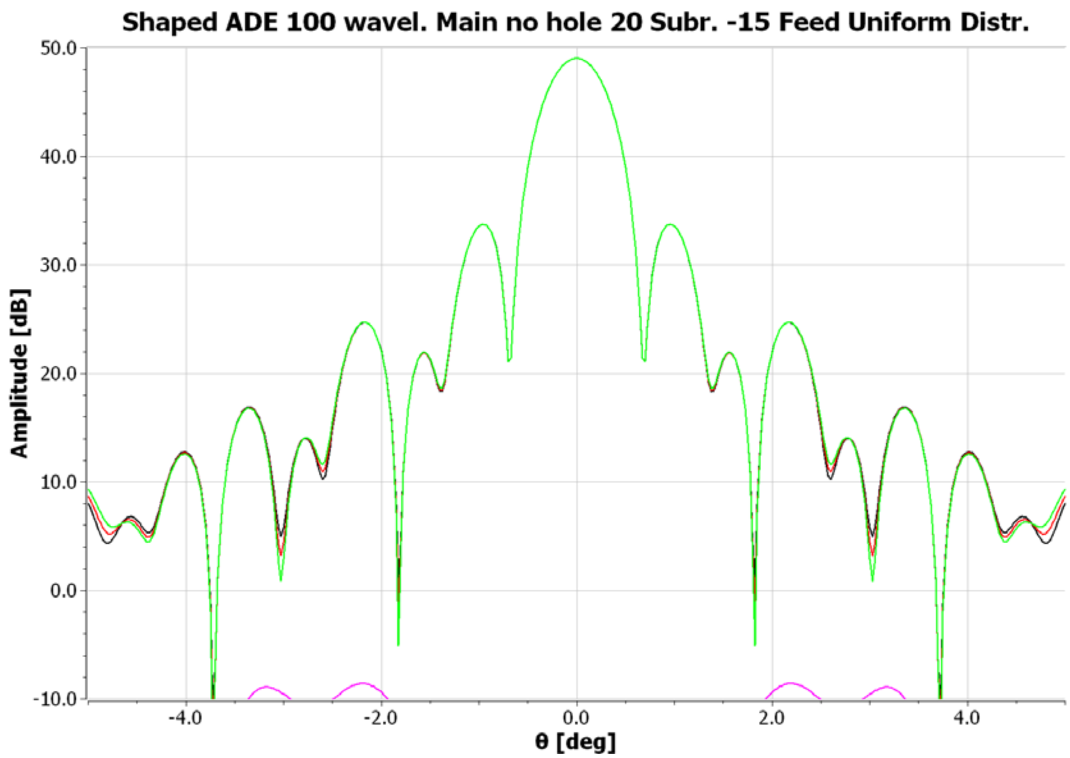

Figure 8-20.5.14 Main Beam Pattern of 100λ diameter Shaped Displaced Axis Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - GRASP Analysis

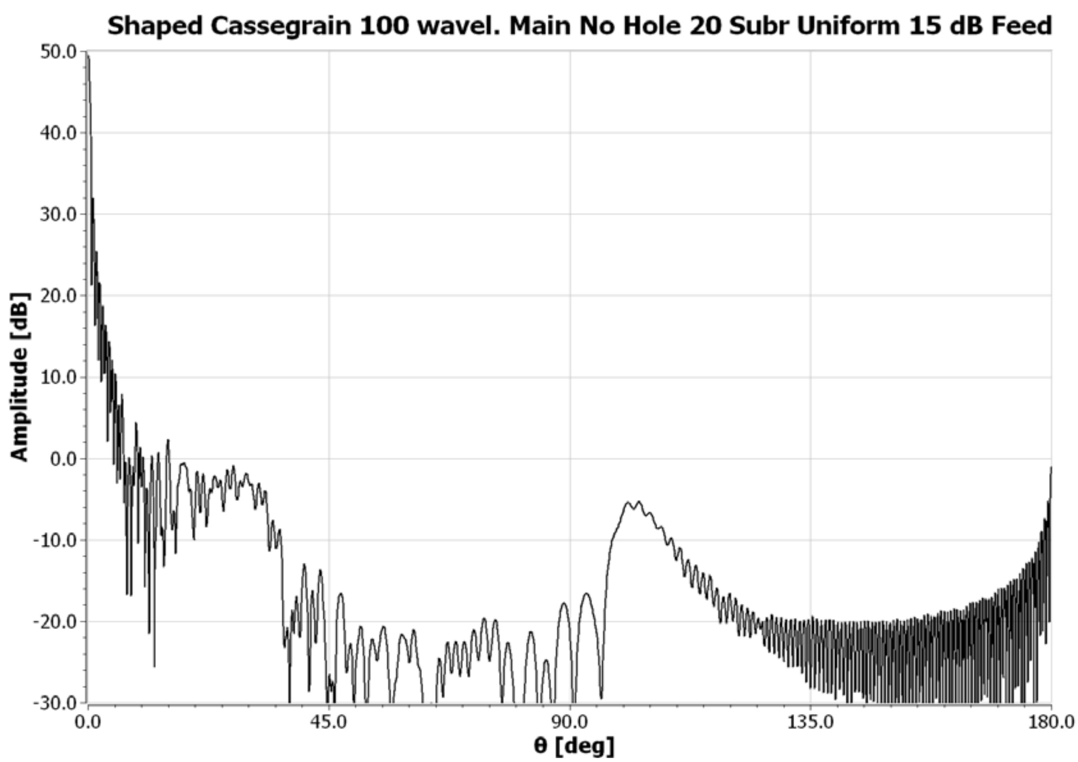

Figure 8-20.5.15 Full Pattern of 100λ diameter Shaped Cassegrain Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - GRASP Analysis

CHAMP Pattern Optimization

CHAMP can optimize the pattern when the two reflectors are added as splines using variables into the geometry.tor. Figures 17 and 18 show the result of an optimization to control the sidelobes of a satellite ground station.

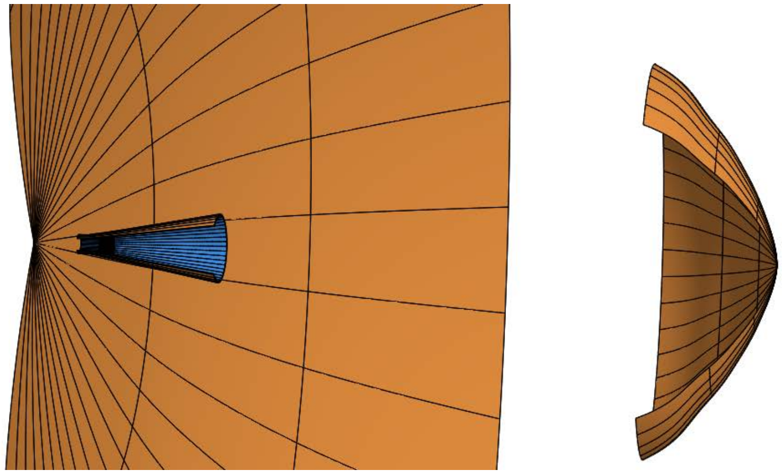

Figure 8-20.5.16 Shaped Reflectors of 100λ diameter Shaped Gregorian Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - CHAMP Analysis

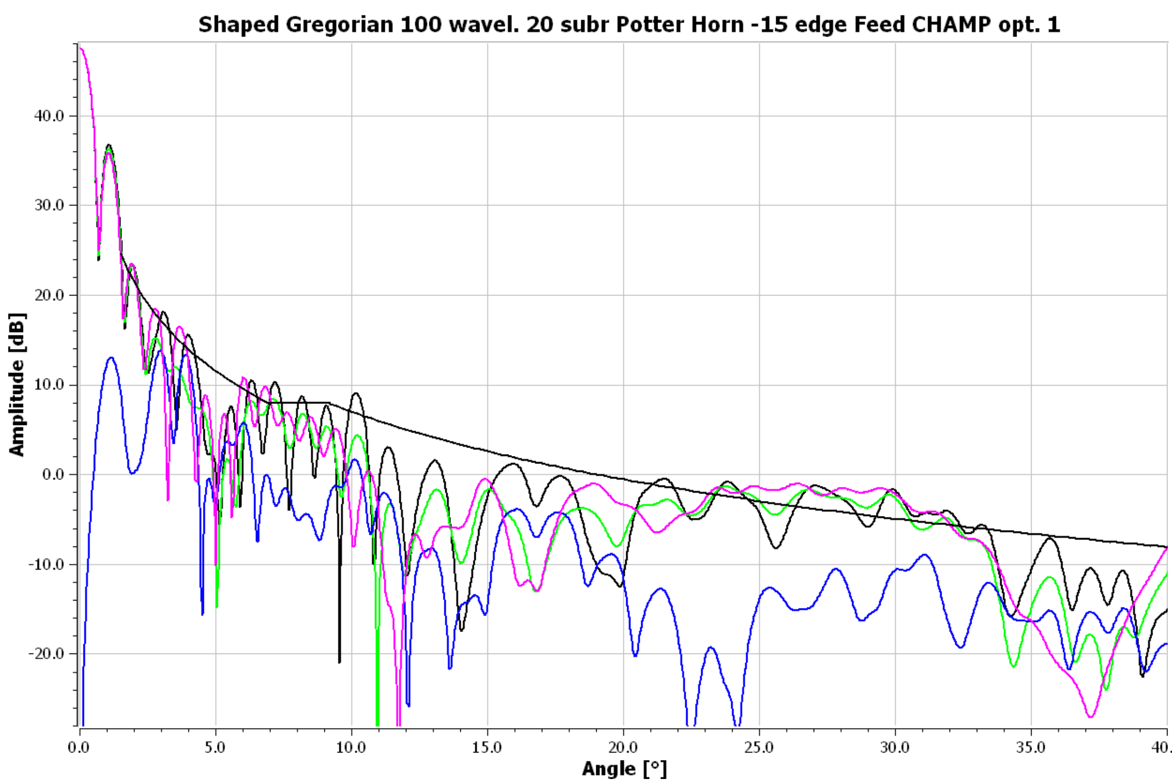

Figure 8-20.5.17 Mainbeam Pattern of 100λ diameter Shaped Gregorian Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - CHAMP Analysis

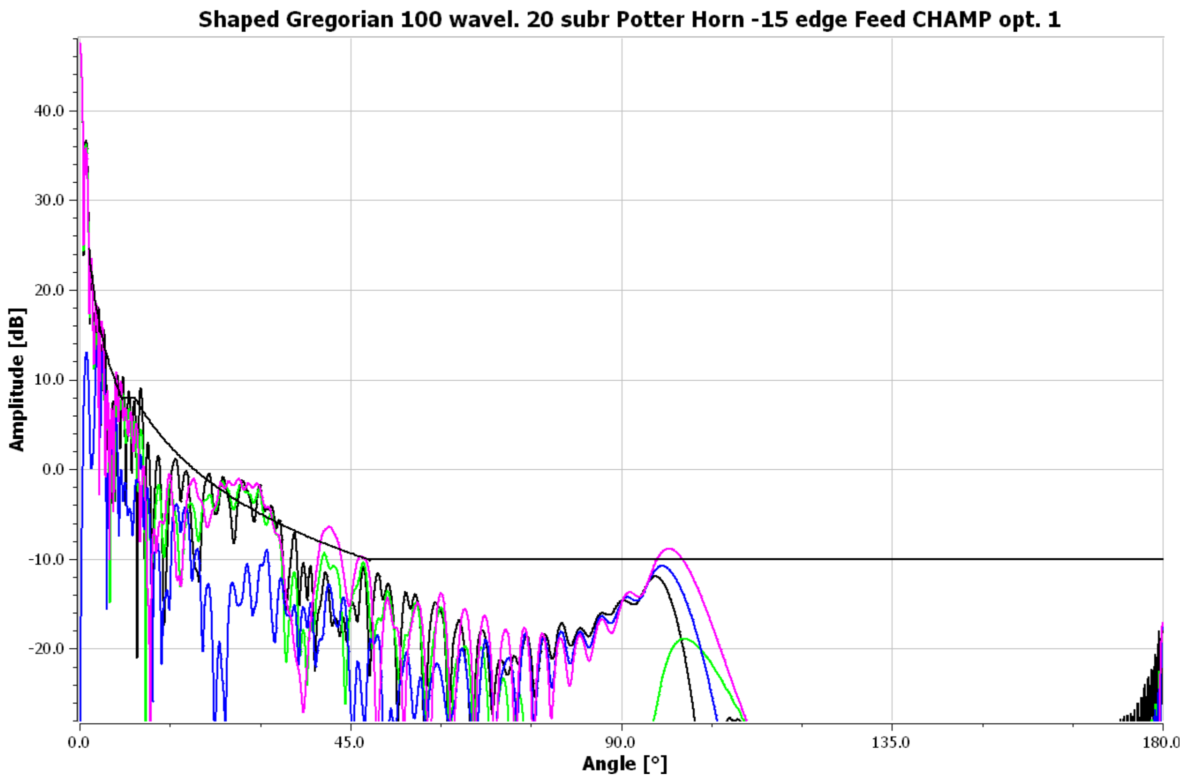

Figure 8-20.5.18 Full Pattern of 100λ diameter Shaped Gregorian Reflector 20λ subreflector and No main hole using -15 dB Feed Taper - CHAMP Analysis

Another approach to optimize a dual reflector using CHAMP is to start with normal dual reflectors that have not been shaped. The program GRANETDS, which can be downloaded from the Cassegrain or Gregorian webpage, generates splines using variables for input to CHAMP. The displaced axis reflector webpage has downloads of the programs DAXSG and DAXSC that also generate CHAMP spline inputs using optimization variables.

Tapered Aperture Distributions

The programs DUSREF and DUSADE can shape the dual reflectors to generate circular Gaussian and circular Taylor aperture distributions. The central blockage will alter the expected aperture pattern and increase the sidelobes.

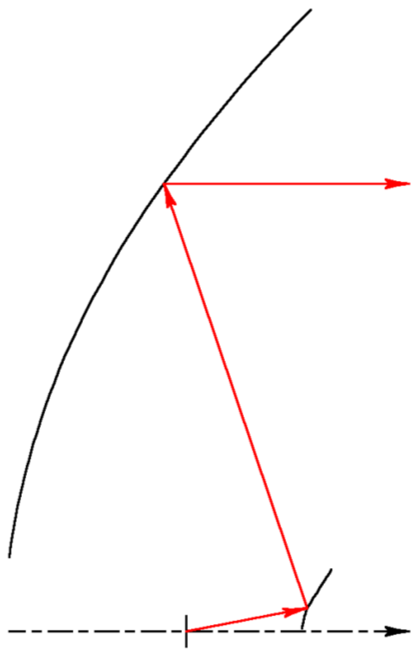

Figure 8-20.5.19 Shaped Dual Cassegrain Type Reflectors to Generate 30 dB Circular Taylor Distribution

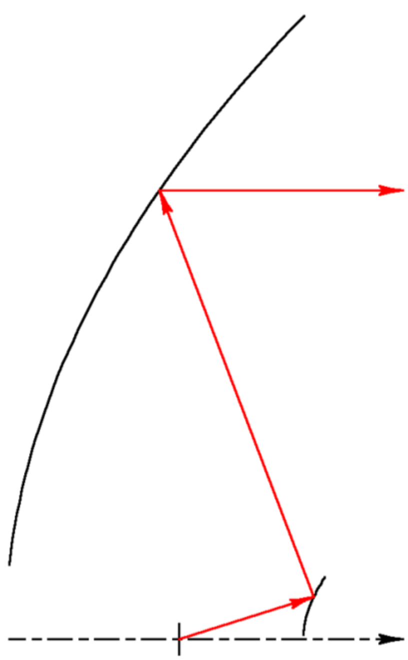

Figure 8-20.5.20 Shaped Dual Cassegrain Type Reflectors to Generate -15 dB Edge Tapered Circular Gaussian Distribution (~ 30 dB first sidelobes)

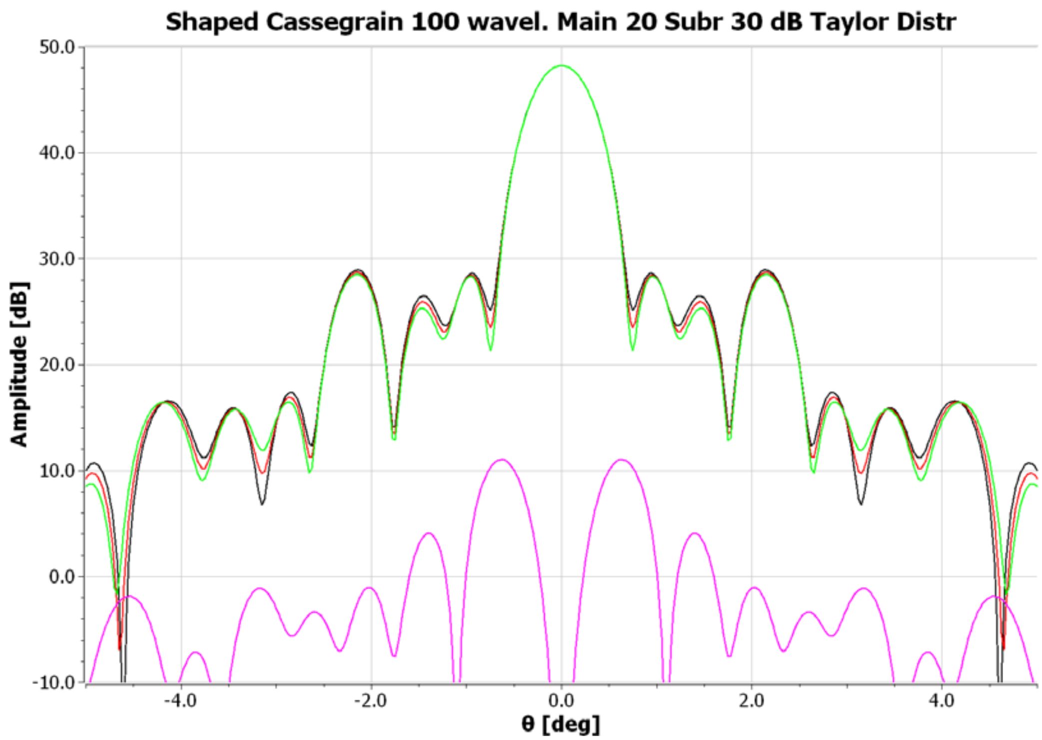

Figure 8-20.5.21 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 10λ subreflector and main hole using -10 dB Feed Taper 30 dB Taylor Distribution - GRASP Analysis

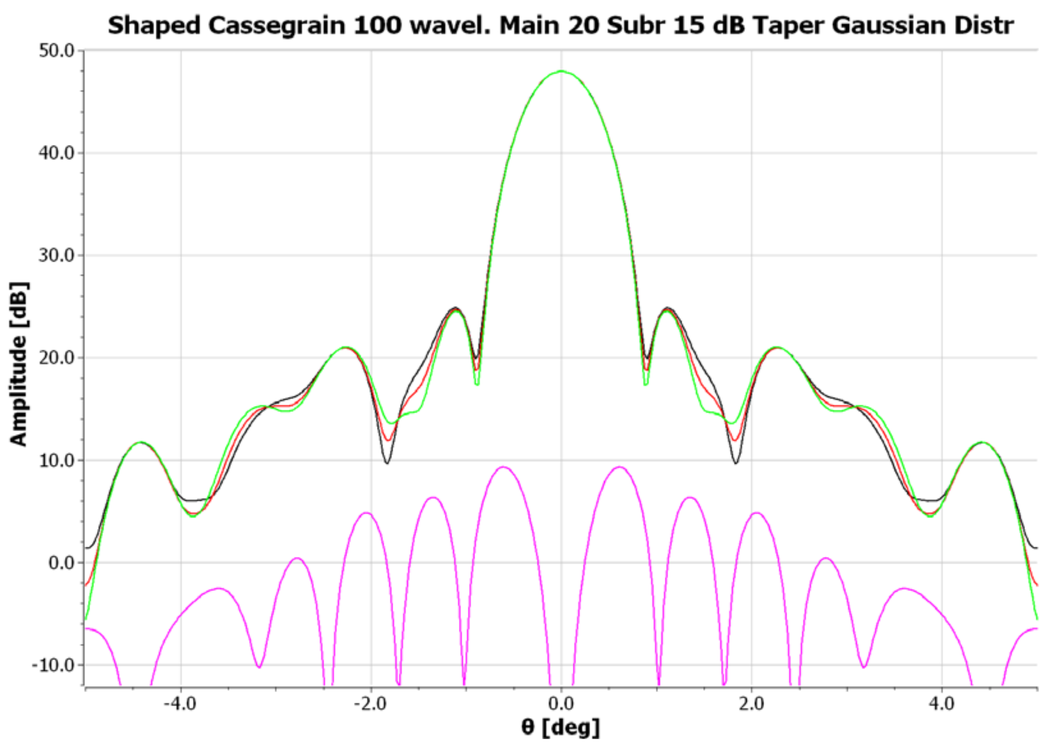

Figure 8-20.5.22 Main Beam Pattern of 100λ diameter Shaped Cassegrain Reflector 10λ subreflector and main hole using -10 dB Feed Taper -15 dB Edge Tapered Gaussian Distribution - GRASP Analysis