# 4-30 Circular Aperture Boresight Axis Near-Field

The Fresnel approximation of fields can be used to compute the near-field power density of an aperture radiator and used to determine radiation safety. While a computation of the radiation from currents excited on a reflector surface or elements in an array can be calculated using the dyadic Greens function, this aperture method produces similar results from a simple integral of over the aperture distribution after being normalized to the far-field level. Assume an axisymmetric aperture voltage distribution and compute the radiation on the axis a distance z using the integral.

$$\int_0^{a_t} E_t(\rho_t)\frac{e^{-jkB}}{B}\rho_t d\rho_t \text{ where } B=\sqrt{\rho_t^2+z^2}$$

The aperture radius is $a_r$. The Fresnel approximation uses $1/z$ for amplitude and $B = z + \rho_t^2/(2z)$ for the exponential phase term. Power density multiplier *PD* is normalized to the far-field distance $2D^2/\lambda$ by normalizing the aperture integral to a radius = 1, using the normalized aperture distribution *E(r)*, and using the normalizing variable $\Delta = z/(2D^2/\lambda)$ which causes the Fresnel approximation of the on-axis power density multiplier to be [1]:

$$PD(\Delta)=\frac{C}{\Delta^2}\left|\int_0^1 E(r)\exp\left[\frac{j\pi}{8\Delta}\left(1-r^2\right)\right]rdr\right|^2 \qquad (4\text{-}30.1)$$

Where *C* is the normalizing constant found from using *PD(1) = 1*, at the far-field distance $2D^2/\lambda$.

$$1/C=\left|\int_0^1 E(r)\exp\left[\frac{j\pi}{8}\left(1-r^2\right)\right]rdr\right|^2 \qquad (4\text{-}30.2)$$

We use numerical integration to compute the constant *C* computed and the power density multiplier at each *Δ* using Eq. (4-30.2) and Eq. (4-30.1).

We compute far-field power density,
$S=P_{in}G/(4\pi z^2)=P_{in}(\pi D/\lambda)^2(ATL)/(4\pi z^2)=P_{in}\pi D(ATL)/(4\lambda z^2)$

*ATL* is the amplitude taper efficiency, Eq. (4-2) or Eq. (4-8) for the circular aperture. The reference distance is $z=2D^2/\lambda$ when substituted above gives the reference power density $S_0=\pi P_{in}(ATL)/(8\lambda D)$ which we multiply *PD* to give the radiated power density in the near field. This ignores the small phase error efficiency, *PEL*, of the quadratic phase error of the circular aperture, Section 4-24 at the normalized phase factor: 1/16 cycles for $z=2D^2/\lambda$, a small factor (~0.04 dB for a parabolic reflector).

[1] R. C. Hansen, ed., *Microwave Scanning Antennas, vol. 1,* Peninsula Publishing, 1985, p. 37.

Consider a prime focus reflector operating at 10 GHz which can be approximated by a circular Gaussian distribution with -12 dB edge amplitude taper for gains 30 dB, 35 dB, and 40 dB.

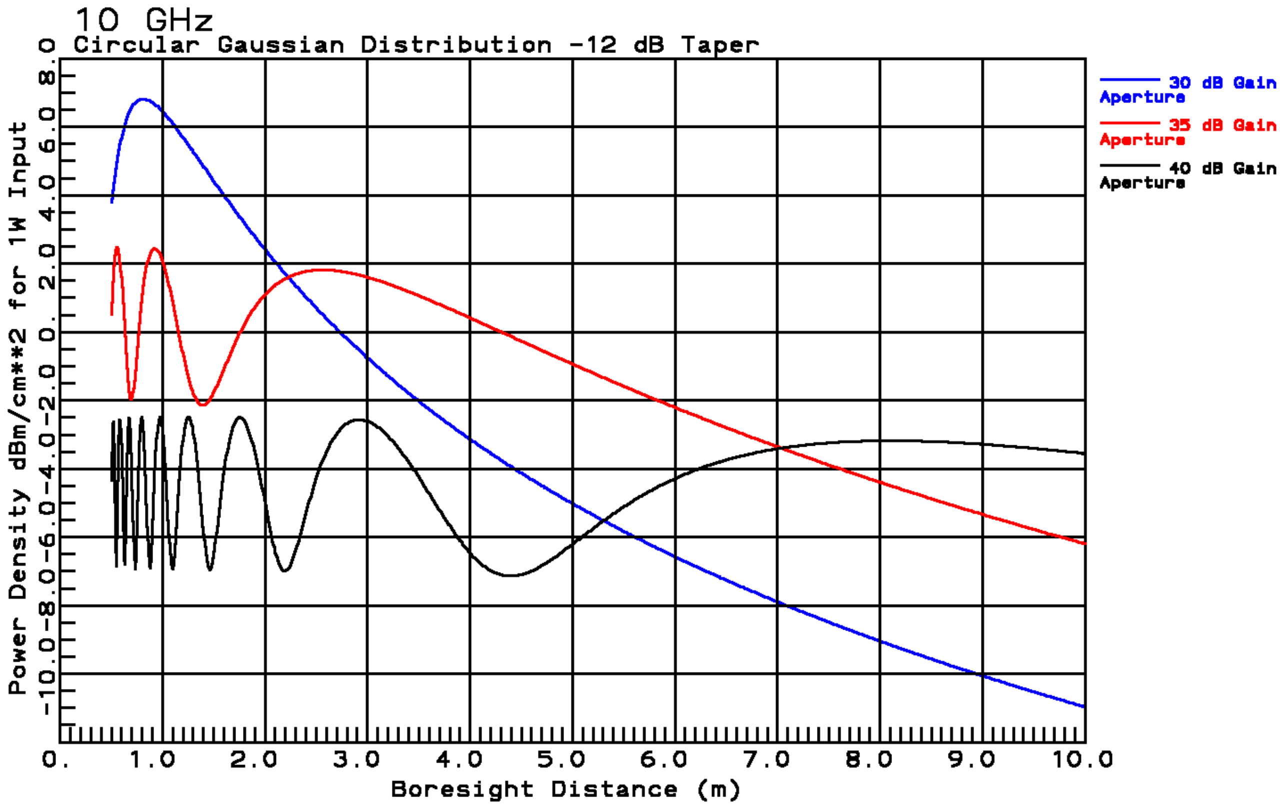


**Figure 4-30.1** Power density radiated from prime focus reflector in Fresnel near-field with 1W input

We compute the reflector diameter for a given gain by adding the amplitude taper loss = 10 Log(*ATR*) to the desired gain $D = \lambda(10^{(Gain(dB)-10Log(ATL))/10}) / \pi$ . The circular Gaussian distribution has -0.62 dB amplitude taper loss for a 12 dB edge taper, Eq. (4-87). At 10 GHz the reflector has diameters: 32.4 cm (30 dB), 57.7 cm (35 dB), and 102.5 cm (40 dB). The input power spreads over a larger area when the reflector diameter increases and the power density in the near-field decreases. Figure 4-30.1 starts at the same distance for all three reflectors and the larger gain reflectors show more near-field cycles. In the extreme near-field the Fresnel approximation fails.

## Coupling between Apertures

The notion of Eq. (1-54a) for coupling between antennas can be used with apertures where we integrate over apertures where each incremental aperture radiation of the transmit aperture is received by a facing axial aperture. The resulting integral is normalized to the power in the each aperture as though it was radiating.

$$\frac{\left|\int_0^{a_T}\int_0^{a_R}\int_0^{2\pi}\int_0^{2\pi} E_T(\rho_T,\phi_T)E_R(\rho_R,\phi_R)\frac{e^{-jkr}}{kr}\rho_T\rho_R d\phi_R d\phi_T d\rho_R d\rho_T\right|^2}{\lambda^2\int_0^{a_T}\int_0^{2\pi}\left|E_T(\rho_T,\phi_T)\right|^2\rho_T d\phi_T d\rho_T\int_0^{a_R}\int_0^{2\pi}\left|E_R(\rho_R,\phi_R)\right|^2\rho_R d\phi_R d\rho_R}$$

where $r$ is the distance between aperture points in cylindrical coordinates

$r = \left|(0,\rho_T,\phi_T)-(z,\rho_R,\phi_R)\right|$ and $k = 2\pi/\lambda$

Combining ATL and PEL and using Fresnel approximation which reduces the exponential phase factor to

$$\left[\rho_T^2+\rho_R^2-2\rho_T\rho_R\cos(\phi_T-\phi_R)\right]/(2z)$$

If the distributions are limited to circularly symmetric the integration over angles reduce to a Bessel function $J_0$ and the efficiency to coupling in the Fresnel zone reduces for aperture radii $a_T$ and $a_R$ to

$$\frac{4\left|\int_0^{a_T}\int_0^{a_R} E_T(\rho_T)E_R(\rho_R)J_0(k\rho_T\rho_R/z)\exp(-k(\rho_T^2+\rho_R^2)/(2z))\rho_T\rho_R d\rho_R d\rho_T\right|^2}{a_T^2a_R^2\int_0^{a_T}\left|E_T(\rho_T)\right|^2\rho_T d\rho_T\int_0^{a_R}\left|E_R(\rho_R)\right|^2\rho_R d\rho_R (ATR)_R(ATR)_T} \quad (4\text{-}30.3)$$

The ideas above have been incorporated in the executable ANTCPP useful for computing Fresnel and Fraunhofer zone power densities, electric fields, and power transfer between facing circularly symmetric aperture distributions given gain (or diameter) of apertures, input power, and distance.

Using two 30 dB gain apertures operating at 10 GHz with 1W input power, ANTCPP computed the following.

```
Axial Coupling between Circular Aperture Antennas

Input Power:   30.00 dBm
Integration Order:  32
Cir. Gaussian Distr. edge taper:  -12.00
Aperture amplitude Loss dB:   -0.62

Transmit Aperture Gain (dB):          30.00

Receive Aperture Gain (dB):           30.00

Frequency (GHz):   10.000
Transmit Aperture Diameter:  0.324 m.
Receive Aperture Diameter:   0.324 m.

             Power      Electric     Receive  Efficiency
Distance    Density      Field       Antenna    dB
   m.      dBm/cm**2     dBV/m        dBm

    0.20       4.99      40.75        29.90   -21.63
    0.40       3.53      39.29        29.79   -15.72
```

| | | | | |
|---|---|---|---|---|
| 0.60 | 5.71 | 41.48 | 29.70 | -12.29 |
| 0.80 | 6.82 | 42.58 | 29.56 | -9.93 |
| 1.00 | 6.45 | 42.21 | 29.42 | -8.13 |
| 2.00 | 2.41 | 38.17 | 28.29 | -3.24 |
| 3.00 | -0.76 | 35.00 | 26.50 | -1.51 |
| 4.00 | -3.14 | 32.62 | 24.68 | -0.83 |
| 5.00 | -5.02 | 30.74 | 23.06 | -0.51 |
| 6.00 | -6.57 | 29.19 | 21.66 | -0.33 |
| 7.00 | -7.89 | 27.87 | 20.43 | -0.22 |
| 8.00 | -9.04 | 26.72 | 19.34 | -0.15 |
| 9.00 | -10.06 | 25.70 | 18.36 | -0.10 |
| 10.00 | -10.97 | 24.79 | 17.48 | -0.07 |
| 15.00 | -14.51 | 21.25 | 14.03 | 0.00 |
| 20.00 | -17.01 | 18.75 | 11.53 | 0.00 |
| 25.00 | -18.95 | 16.81 | 9.59 | 0.00 |
| 30.00 | -20.53 | 15.23 | 8.01 | 0.00 |

At the close distance of 0.2 m the coupling between apertures is only down 0.1 dB and the 30 dBm input power is almost completely collected by the receiving aperture. When the spacing is 15 m, the far-field coupling can be found from path loss Eq. (1-8) or (1-9) -15.97 dB or a received power of 14.03 dBm. When the distance doubles to 30 m, both the power density and received power drop by 6.02 dB.

Suppose we increase the gain (and diameter) of the receive aperture to 40 dB (1.025 m) then the coupling will change.

```
Axial Coupling between Circular Aperture Antennas

Input Power:   30.00 dBm
Integration Order:  32
Cir. Gaussian Distr. edge taper:  -12.00
Aperture amplitude Loss dB:   -0.62

Transmit Aperture Gain (dB):          30.00

Receive Aperture Gain (dB):           40.00

Frequency (GHz):   10.000
Transmit Aperture Diameter:  0.324 m.
Receive Aperture Diameter:   1.025 m.
```

| Distance m. | Power Density dBm/cm**2 | Electric Field dBV/m | Receive Antenna dBm | Efficiency dB |
|---|---|---|---|---|
| 0.20 | 4.99 | 40.75 | 23.52 | -38.01 |
| 0.40 | 3.53 | 39.29 | 23.86 | -31.65 |
| 0.60 | 5.71 | 41.48 | 23.69 | -28.30 |
| 0.80 | 6.82 | 42.58 | 23.67 | -25.82 |
| 1.00 | 6.45 | 42.21 | 23.71 | -23.85 |

| | | | | |
|---|---|---|---|---|
| 2.00 | 2.41 | 38.17 | 23.74 | -17.80 |
| 3.00 | -0.76 | 35.00 | 23.45 | -14.56 |
| 4.00 | -3.14 | 32.62 | 23.29 | -12.22 |
| 5.00 | -5.02 | 30.74 | 23.23 | -10.34 |
| 6.00 | -6.57 | 29.19 | 23.64 | -8.35 |
| 7.00 | -7.89 | 27.87 | 24.09 | -6.56 |
| 8.00 | -9.04 | 26.72 | 24.32 | -5.17 |
| 9.00 | -10.06 | 25.70 | 24.33 | -4.14 |
| 10.00 | -10.97 | 24.79 | 24.18 | -3.37 |
| 15.00 | -14.51 | 21.25 | 21.23 | -2.80 |
| 20.00 | -17.01 | 18.75 | 19.41 | -2.12 |
| 25.00 | -18.95 | 16.81 | 17.79 | -1.81 |
| 30.00 | -20.53 | 15.23 | 16.37 | -1.64 |
| 35.00 | -21.87 | 13.89 | 15.14 | -1.53 |
| 40.00 | -23.03 | 12.73 | 14.05 | -1.47 |
| 45.00 | -24.06 | 11.70 | 13.07 | -1.42 |
| 50.00 | -24.97 | 10.79 | 12.19 | -1.39 |
| 55.00 | -25.80 | 9.96 | 11.38 | -1.36 |
| 60.00 | -26.55 | 9.21 | 10.65 | -1.34 |
| 70.00 | -27.89 | 7.87 | 9.33 | -1.32 |
| 80.00 | -29.05 | 6.71 | 8.19 | -1.30 |
| 90.00 | -30.08 | 5.68 | 7.18 | -1.29 |
| 100.00 | -30.99 | 4.77 | 6.27 | -1.28 |
| 110.00 | -31.82 | 3.94 | 5.45 | -1.27 |
| 120.00 | -32.58 | 3.18 | 4.70 | -1.27 |
| 130.00 | -33.27 | 2.49 | 4.01 | -1.27 |
| 140.00 | -33.91 | 1.85 | 3.37 | -1.26 |
| 150.00 | -34.51 | 1.25 | 4.03 | 0.00 |

The maximum received power reduces by about 6 dB which seems unexpected since the larger aperture of the receive antenna should collect all the radiated power. We need to remember reciprocity. If we switch transmit and receive antennas, the power transferred will remain the same. When the larger aperture (40 dB gain) transmits, the smaller aperture will fail to collect power in the near-field which streams past it.

We can violate the Fresnel zone approximation if we consider points close to the aperture. The program ANTCPP will report higher received power than transmitted in these regions. Both the received power and power density in these regions is incorrect.