4-28 Array Gain Efficiency due to Coupled Elements using GRASP (TICRA)

While this section generates patterns by using the program GRASP, array gain efficiency can be computed using elements computed by using other CEM programs as long as the patterns are converted to GRASP *.cut files. The *.cut file is a simple ASCII format with separated scans in θ and steps in ϕ over the radiation sphere. The program **TICARR** generates *.isp GRASP array geometry file and multiple excitation files: *.exi for GRASP runs using array definition of program **XADEF** (Section 4-27). In an example of 225 element square array **TICARR** uses "Gaussian_beam_90" as the GRASP element pattern. The excitation can include a quadratic phase distribution in both the *x*- and *y*-planes. GRASP computes gain as (number of elements)*(element gain). While this works for widely spaced array elements, it is incorrect for radiators who gain exceeds the radiation gain of a uniformly illuminated area associated with each element.

```
C:\milligan\ANTENNA\ARRAY>ticarr
Enter input 0 keyboard, 1 file 0
Enter XADEF array File Name sq225t.arr
Number of Elements:    225
Frequency:             1.000GHz
Enter Ticra element position output filename (.isp) sq225t.isp
Enter units: 1 in, 2 ft, 3 mm, 4 cm, 5 m 4
Enter label
225 element square array with 30 dB n-bar = 6 Taylor Distributions
Enter TICRA source object name for element pattern gaussian_beam_90
Enter Ticra excitation output filename (.exi) sq22t0.exi
Enter quadratic phase taper (deg) X,Y, Max Radius X,Y 0,0,10,10
Enter array efficiency dB 0
Enter 1 to form beam of array 1
Enter scan direction of array Theta, Phi 0,0
Enter beam pointing frequency (GHz) 1.3
Quantize Phase? n
Enter Feed Error (1 Sigma) Ampl(dB), Phase(deg) 0,0
Another beam excitation file? y
Enter Ticra excitation output filename (.exi) sq225t7.exi
Enter quadratic phase taper (deg) X,Y, Max Radius X,Y 0,0,10,10
Enter array efficiency dB 0
Enter 1 to form beam of array 1
Enter scan direction of array Theta, Phi 7.5,0
Enter beam pointing frequency (GHz) 1.3
Quantize Phase? n
Enter Feed Error (1 Sigma) Ampl(dB), Phase(deg) 0,0
Another beam excitation file? y
Enter Ticra excitation output filename (.exi) sq225t7-4.exi
Enter quadratic phase taper (deg) X,Y, Max Radius X,Y 0,0,10,10
Enter array efficiency dB 0
Enter 1 to form beam of array 1
Enter scan direction of array Theta, Phi 7.5,0
Enter beam pointing frequency (GHz) 1.3
Quantize Phase? y
Enter Number of Phase Bits 4
Enter Feed Error (1 Sigma) Ampl(dB), Phase(deg) .5,5
Another beam excitation file? n
Process another array file? n
Stop - Program terminated.
```

The efficiency can be entered here or specified in GRASP. Below a technique using normalized mutual resistances is given to compute a planar array gain. This can be used in the standard PO/PTD analyses. The MOM solutions account for mutual coupling. The program applies phasing across the array elements to point the beam at a particular frequency. Quantized phase is used for digitized (bits) phase shifters. The program also applies random amplitude and phase errors to account for feed network errors. As shown above, multiple excitation files can be created for use with one array in GRASP.

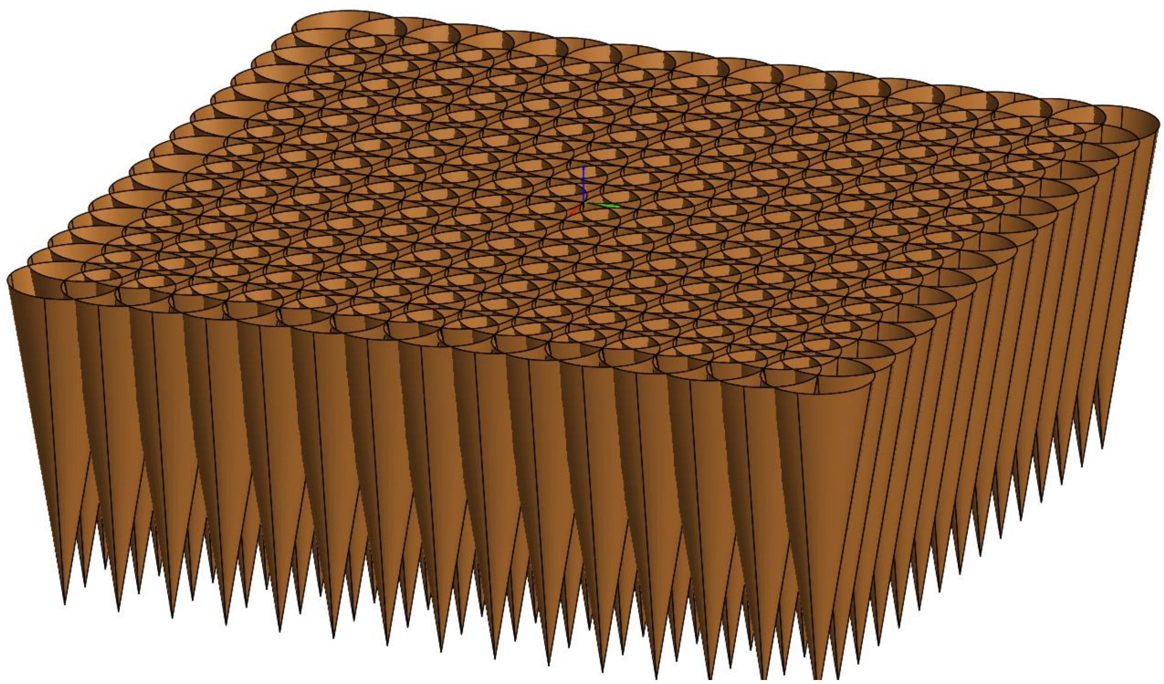

Figure 1 Square 225 element Array 90 deg. BW Gaussian Beam illustrated in GRASP

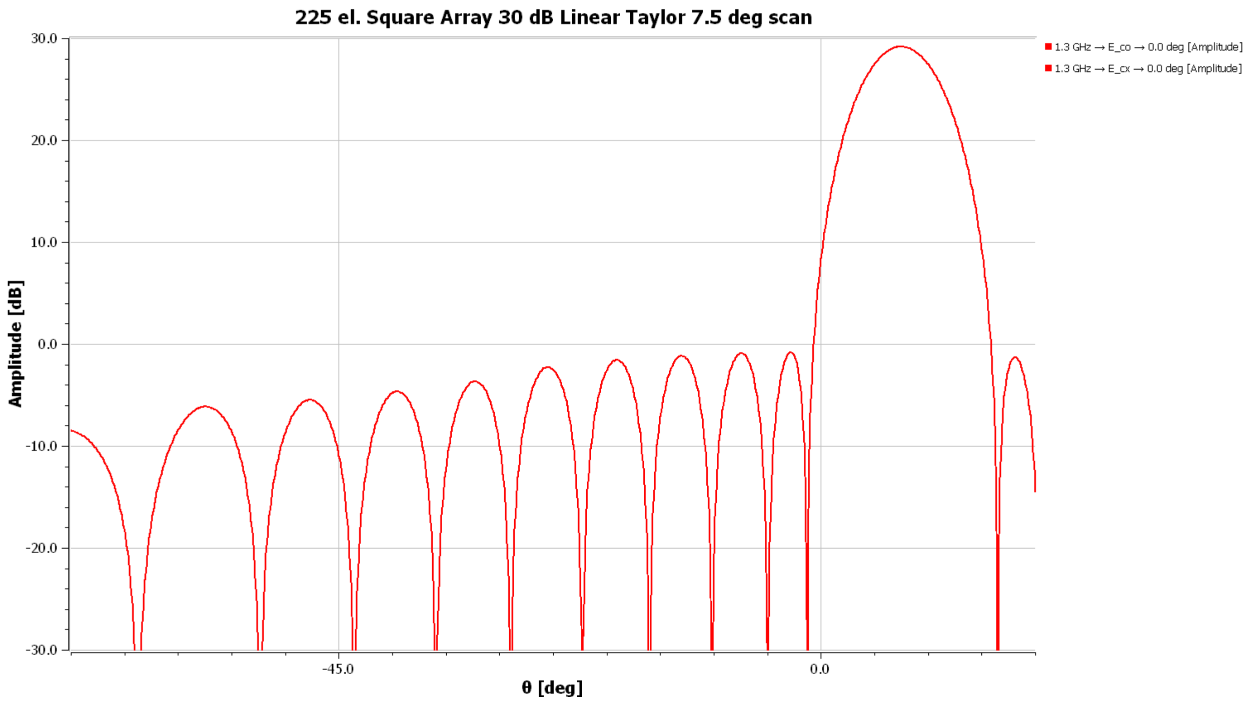

Figure 2 15 element 30 dB Linear Taylor Distribution Array Response in GRASP

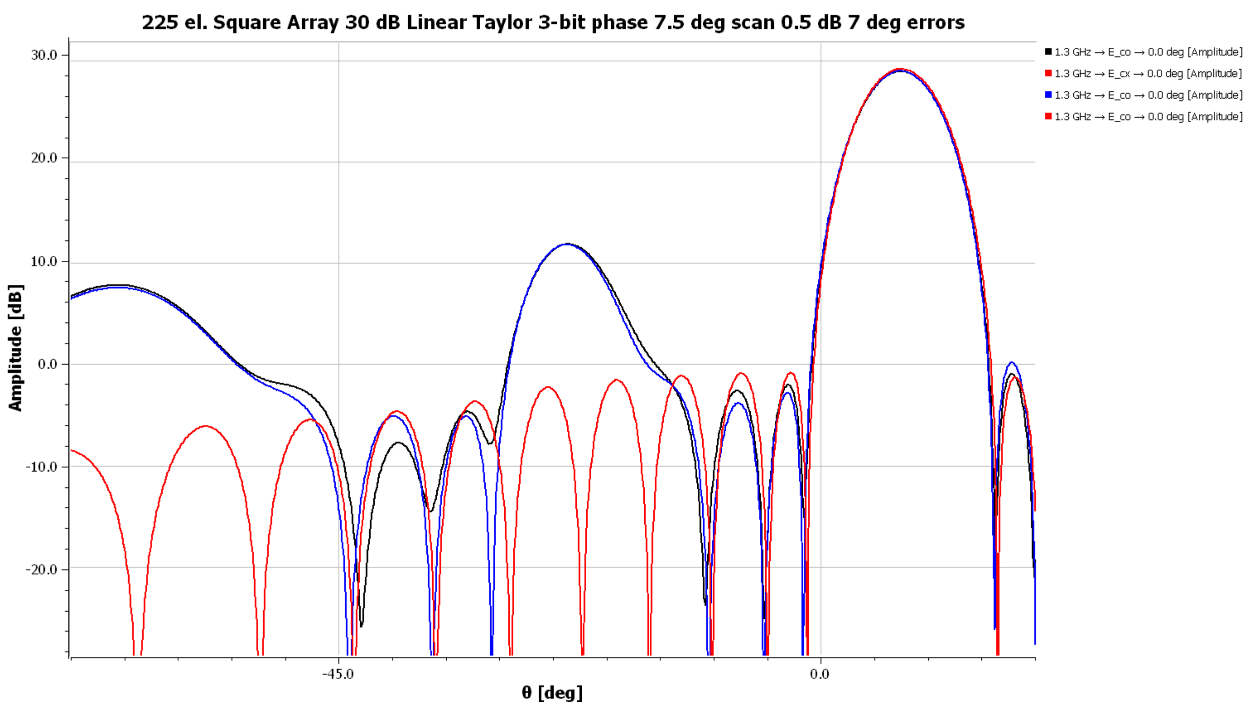

Figure 3 15 element 30dB Linear Taylor Distribution 3-bit Phase Shifters 7.5° scan

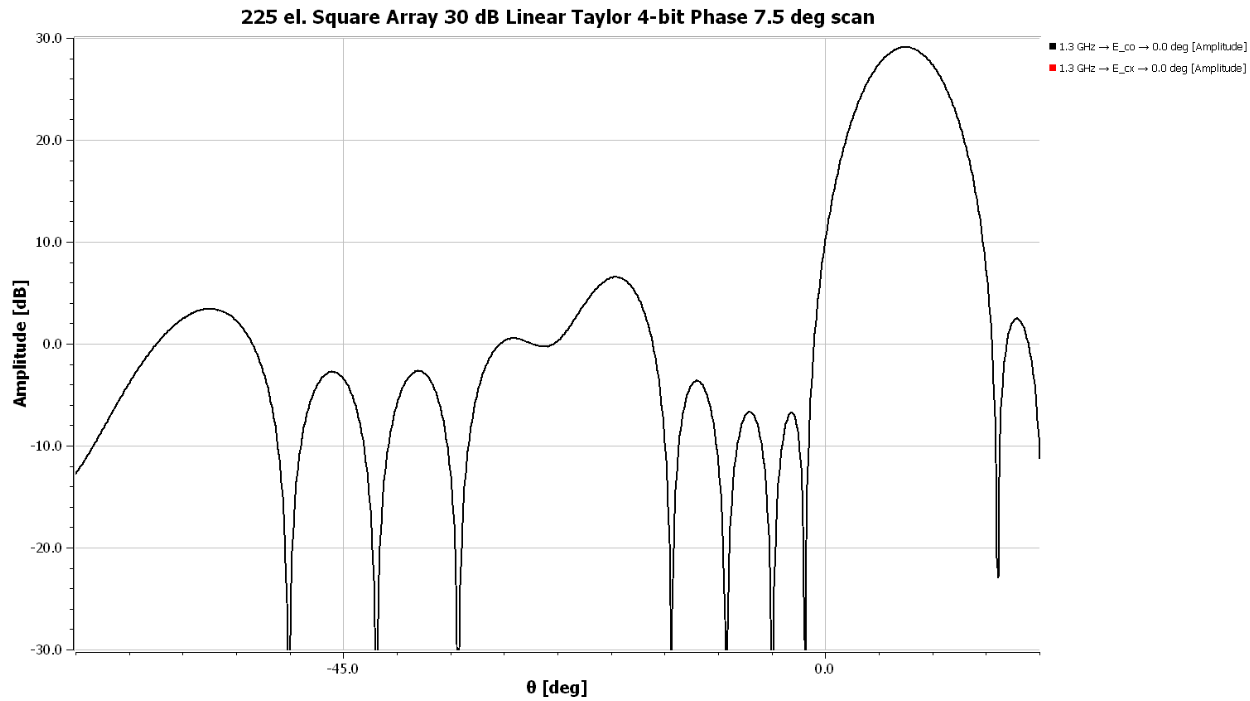

Figure 4 15 element 30dB Linear Taylor Distribution 4-bit Phase Shifters 7.5° scan

Spherical Cap Array definition

XADEF was run to generate a spherical cap array whose elements point normal to the sphere and are rotated to the same polarization.

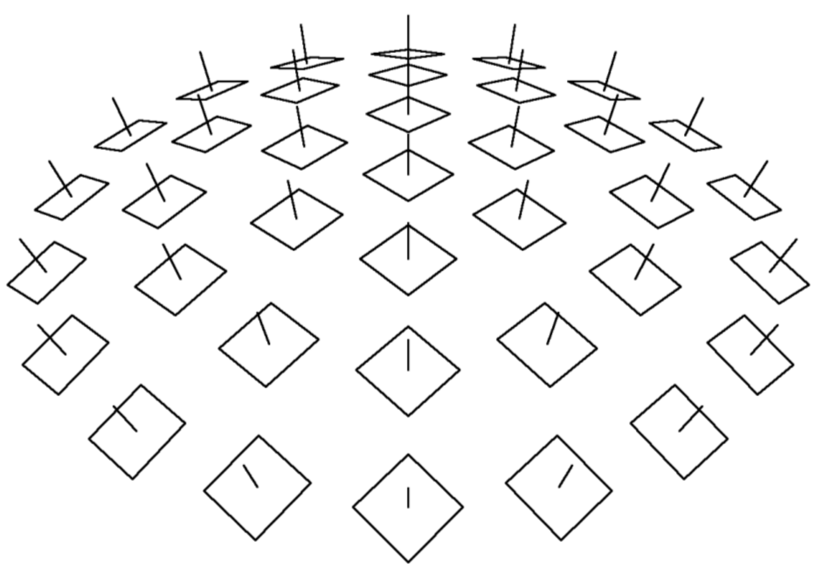

Figure 5 37 element spherical cap array pictured in perspective

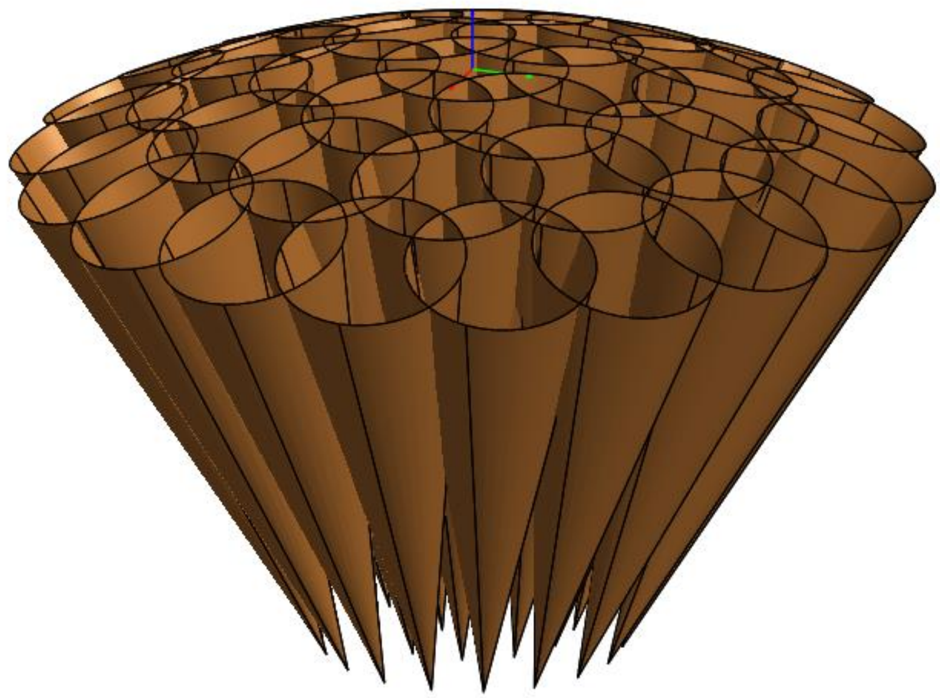

Figure 6 37 element Spherical Cap Array using 90° BW Gaussian Beam elements in GRASP

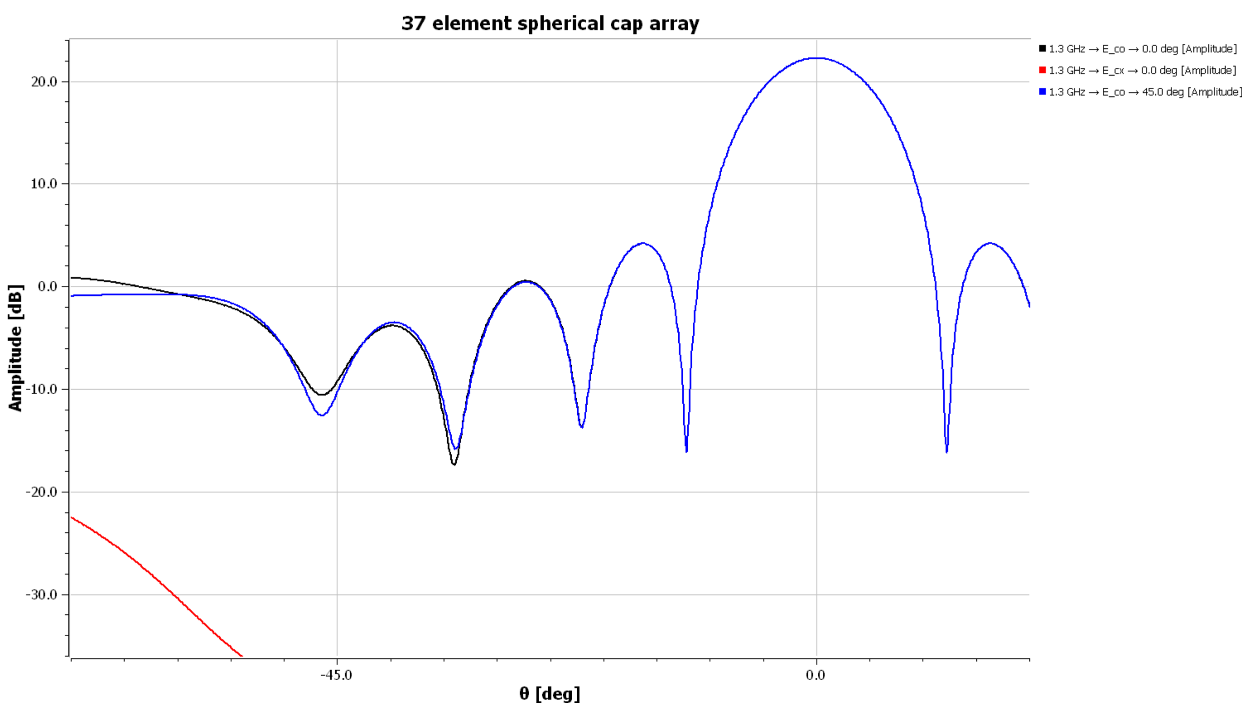

Figure 7 37 element Spherical Cap Array Pattern phased by TICARR to form a beam

Computation of Planar Array Directivity using Normalized Mutual Resistance

Section 3-12 discusses using normalized mutual resistance between two elements in an array to calculate the total input power to an array for a given amplitude distribution exciting a planar array.

$$\frac{R_{12}(x)}{R_{11}} = \frac{\text{(element directivity)}}{2\pi}\int_0^{2\pi}\int_0^{\pi} E_e^2(\theta,\phi)\cos^2\left(\frac{\pi x}{\lambda}\cos\phi\sin\theta\right)\sin\theta d\theta d\phi - 1 \qquad (3\text{-}29)$$

As element spacing *x* increases the $\cos^2\left(\frac{\pi x}{\lambda}\cos\phi\sin\theta\right)$ term oscillates rapidly and is difficult to evaluate numerically.

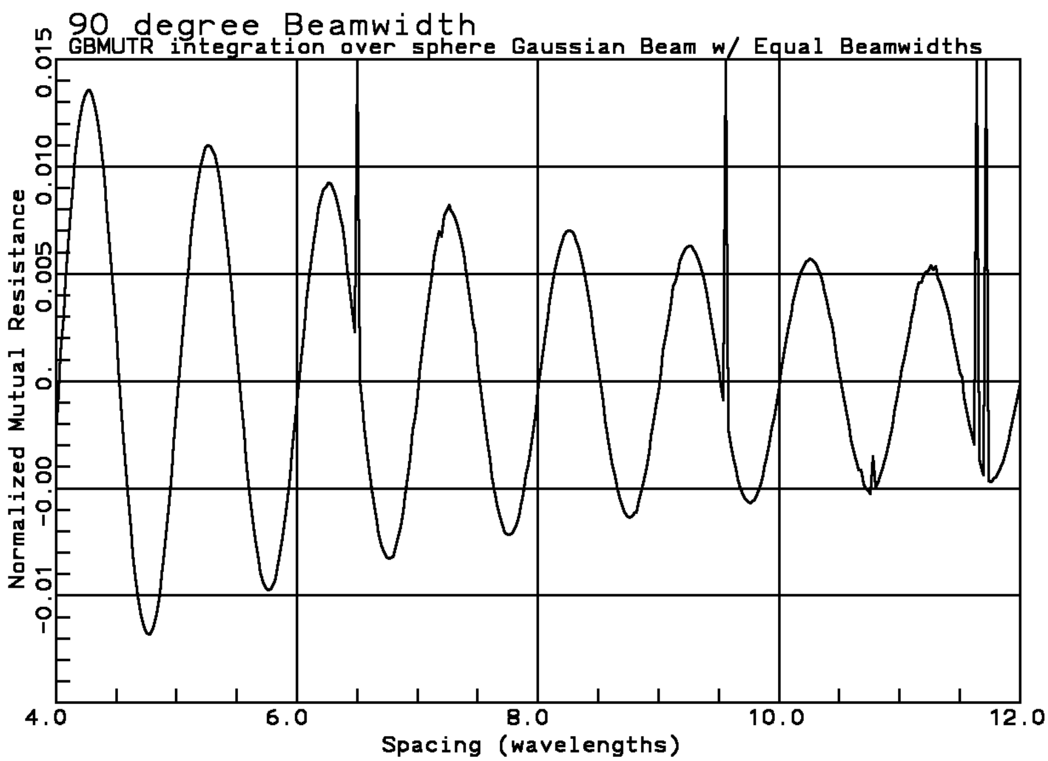

Figure 8 Normalized mutual resistances for equal 90° beamwidth Gaussian beam elements using single precision numerical integration

The Phi integration can be reduced to a Bessel function for an axisymmetric pattern antenna.

$$\int_0^{2\pi}\cos^2(z\cos\varphi)d\varphi = \pi\left[J_0(z)+1\right]$$

Using this Bessel function for equal beamwidth elements the narrow spikes can be removed from computations of normalized mutual resistance between two elements.

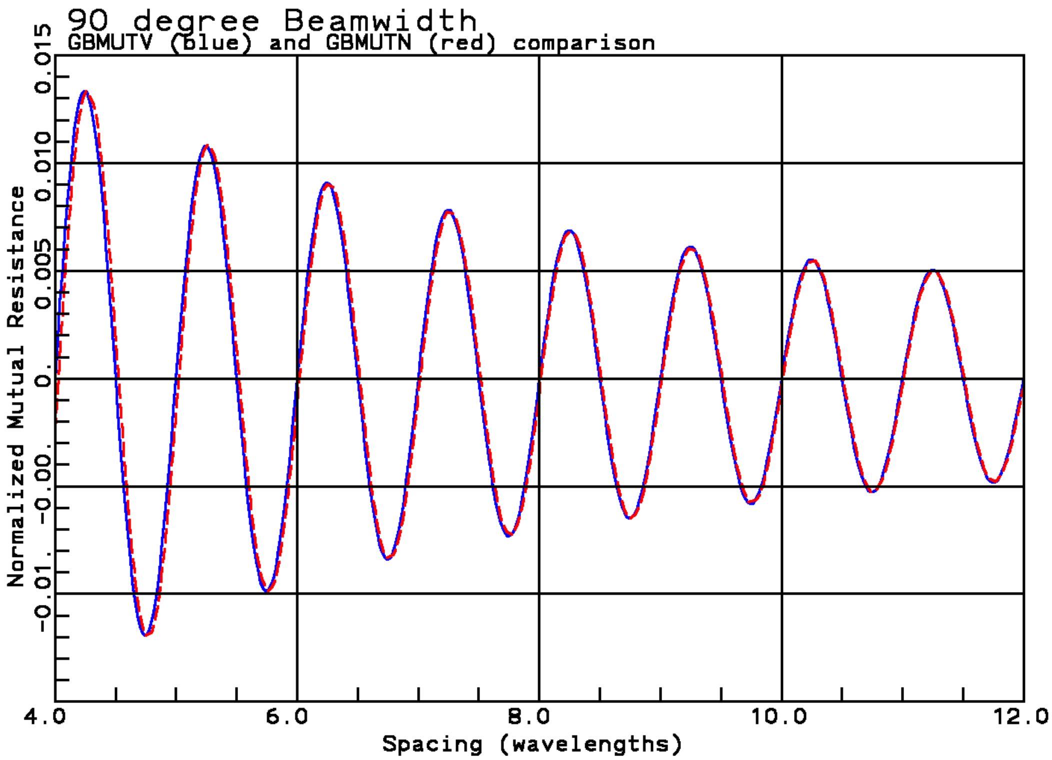

Figure 9 Normalized mutual resistances for equal 90° beamwidth Gaussian beam elements using Bessel function instead of numerical integration. The blue curve shows using vector effective height for normalized mutual resistances

$$Z_{12} = \frac{(V_2)_{OC}}{I_1} = \frac{jk\eta e^{-jkr}}{4\pi r}\mathbf{h}_1 \cdot \mathbf{h}_2 \qquad (1\text{-}51)$$

Compute the power gain *G* of the antenna at 90° in direction *φ* between the two identical antennas and using the spacing between them *S*, Eq. (1-51) reduces to the following expression for normalized mutual impedance.

$$Z_{12} = jG\frac{\cos(2\pi S/\lambda) - j\sin(2\pi S/\lambda)}{2\pi S/\lambda}$$

Figure 9 shows the use of mutual impedance computed by using Eq. (1-51) of the vector effective height of the Gaussian beam antennas at θ = 90°. Equation (1-51) fails to predict the mutual impedance for closely spaced elements but computes good values for widely spaced antennas. This idea can be used for unequal *E*- and *H*-plane beamwidth antennas to compute normalized mutual impedance at large spacing where numerical integration fails.

Figure 10 illustrates that the normalized mutual resistance between two elements increases for narrow beamwidth (high gain) elements as they are brought closer together. Intuitively narrow beamwidth (higher gain) element couple less than wide beam (low gain elements), but only happens when they are separated sufficiently. When the effective area associated with the gain of an element overlaps the effective area of another antenna, the coupling (mutual resistance) increases rapidly with increased overlap.

Figure 11 shows the variation of normalized mutual resistances with Phi plane for unequal *E*- and *H*-plane beamwidth Gaussian beam antenna coupled pair of antennas. The mutual resistances increase for closely spaced antennas in the plane of the narrow beamwidth compared to the wide beam pattern cut. The ϕ direction of direction of the line between two elements needs to be accounted for when computing the mutual resistance. This requires interpolation between the various ϕ of a given distance.

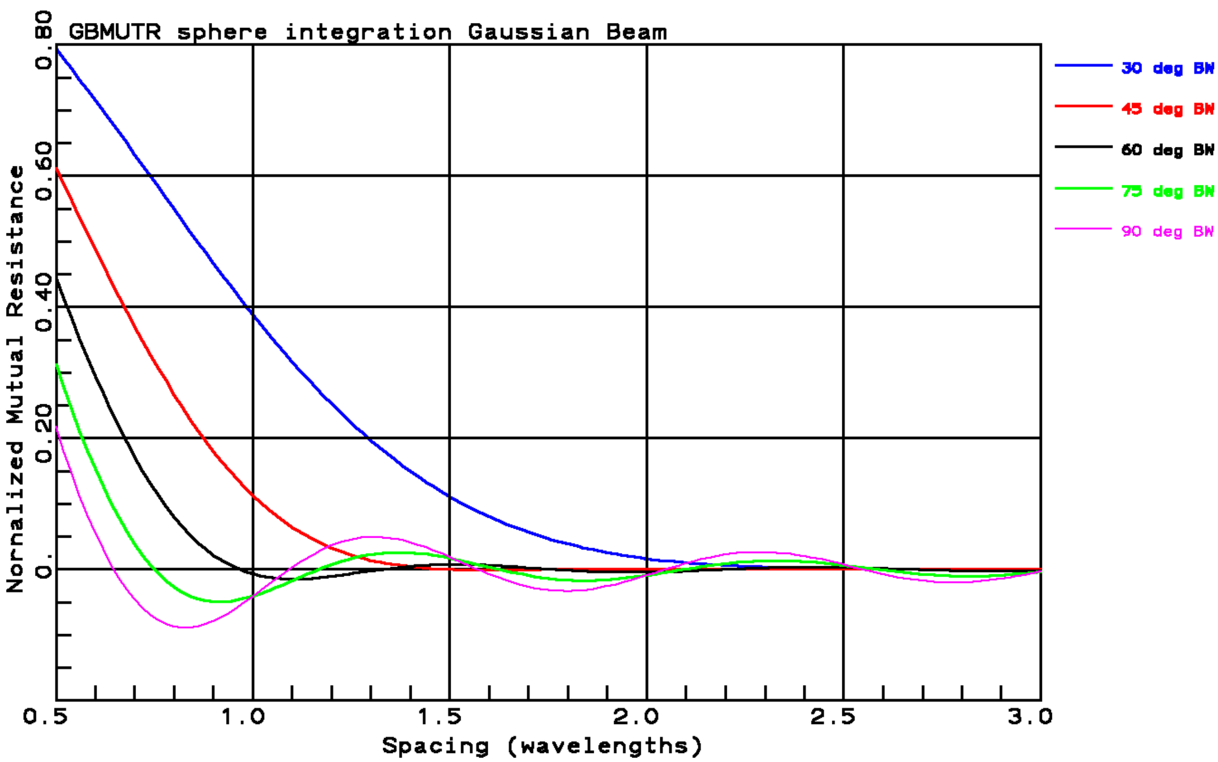

Figure 10 Normalized mutual resistances of closely spaced Gaussian beam antennas

Equation (3-29) presents a problem as the antenna gain increases, such as, horns. A horn fills the effective area associated with the area in the aperture plane of the array so it is impossible to pack them closely. Second, the coupling between side-by-side horns is low. The integral the pattern of the two-element array multiplied by element directivity divided by 2π is approximately one. When we subtract one from the integral expression, the remainder is quite small and we lose significant figures unless we use a double precision Romberg integral evaluation.

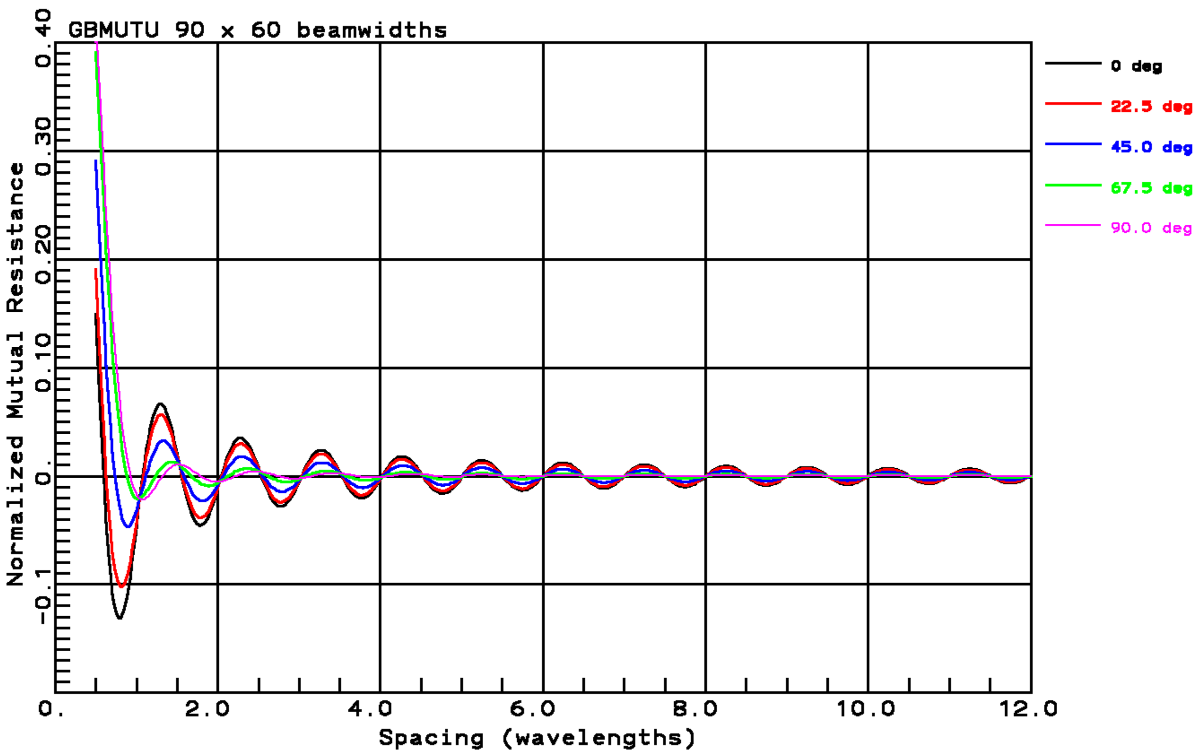

Figure 11 Normalized mutual resistances versus Phi for unequal beamwidth Gaussian beam antennas

Executables to compute Normalized Mutual Resistances

GBMUTN: Gaussian beam with equal beamwidths using Bessel equation

GBMUTU: Gaussian beam with unequal beamwidths for multiple Phi angles between 0° and 90° using combination numerical integration and vector effective heights.

Pattern Approximation 1: $E = \cos^N(\theta/2)$

CBMUTN: Pencil beam with equal beamwidths using Bessel equation

CBMUTU: Pencil beam with unequal beamwidths for multiple Phi angles between 0° and 90° using combination numerical integration and vector effective heights.

Pattern Approximation 2: $E = \cos^N(\theta)$ for $\theta \leq 90°$

CBMUT2N: Pencil beam with equal beamwidths using Bessel equation

By using equation (3-30) we can calculate the directivity of the square 225 element planar array using tables of normalized mutual resistances versus element spacing to compute the total power input to all elements in the array even though power is coupled into nearby elements to be dissipated in the nearby antennas. Figure 12 plots the directivity for the 225-element array excited with 30 dB Taylor distributions in both the *E*- and *H*-planes using constant beamwidth elements.

Executables to compute Array Directivity and Efficiency given XADEF array

GPDIR: Equal beamwidth array elements using data of Normalized Mutual Resistance versus spacing

GPDIRU: Unequal beamwidth array elements using data of Normalized Mutual Resistance versus spacing at multiple Phi angles

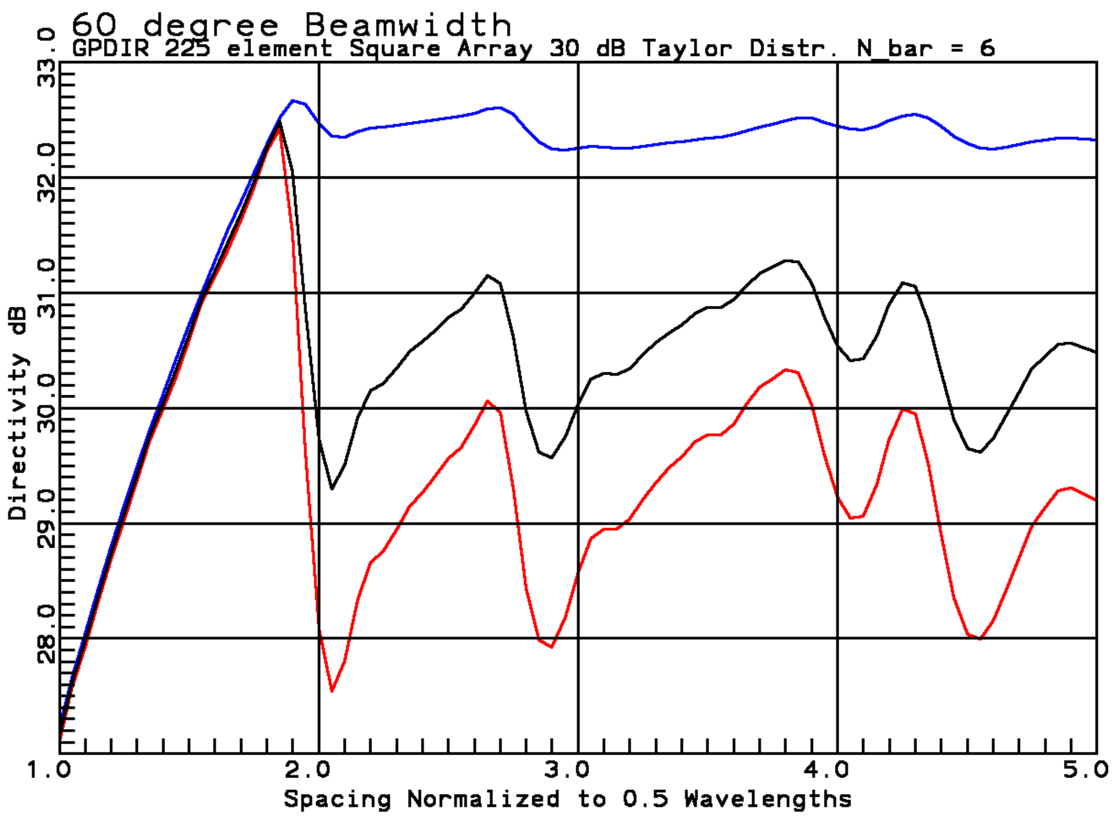

Figure 12 Directivity versus element spacing of 60° (blue), 75° (black), and 90° (red) beamwidth elements

The 60° beamwidth elements illustrate nearly constant directivity (gain) versus spacing after one wavelength spacing. The 75° and 90° beamwidth array elements show a cycling into spacings with grating or partial grating lobes. The lower portion of the curve illustrates the rapid directivity fall off. The higher gain elements (60°) produce the same gain as lower gain (90°) array elements because the array gain is area limited. Later we will see that excess element gain in the closely spaced region leads to blind spots where transmitted power reflects into the element and is lost in internal loads.

Given an element beamwidth of 90°, Table 1 lists directivity and efficiency versus element spacing for 90° beamwidth elements. The array has elements spaced λ/2 and frequency corresponds to spacing ratio because the array was designed to operate at frequency = 1.

Table 1 Array Directivity and Efficiency versus element spacing with Frequency equal spacing ratio on λ/2.

Directivity of Planar Array

XADEF file name: sq225.arr
Mutual Impedance File: gbmutu90.dat
Rotation of beam ellipse: 0.00
Element Directivity = 7.09

Frequency	Directivity	Efficiency
1.000	27.12	-2.15
1.050	27.57	-1.70
1.100	27.93	-1.33
1.150	28.32	-0.94
1.200	28.69	-0.58
1.250	29.02	-0.25
1.300	29.36	0.10
1.350	29.71	0.44
1.400	30.00	0.73
1.450	30.27	1.00
1.500	30.59	1.32
1.550	30.92	1.65
1.600	31.14	1.87
1.650	31.37	2.10
1.700	31.63	2.37
1.750	31.92	2.65
1.800	32.23	2.97
1.850	32.42	3.15
1.900	31.51	2.25
1.950	29.63	0.36
2.000	28.08	-1.18
2.050	27.54	-1.72
2.100	27.80	-1.46
2.150	28.33	-0.94
2.200	28.66	-0.61
2.250	28.76	-0.50
2.300	28.95	-0.32
2.350	29.14	-0.12
2.400	29.27	0.00

To obtain the correct directivity from an array computation (GRASP PO/PTD) the efficiency is entered either in **TICARR** or directly in GRASP. At λ/2 spacing (Frequency = 1, Table 1) for 90° beamwidth elements causes a loss of 2.15 dB from (number of elements)*(element gain).

When we evaluate the planar array efficiency for an equivalent 225 element array fed with uniform amplitude feeding, we get the same efficiencies. This means that the coupling from individual elements to all other elements is unaffected by their excitation. Power is coupled from each element to all its neighbors independent of their feeding amplitude and phasing. The efficiency is unaffected by scanning (phasing) or amplitude distribution to lower sidelobes. The effective pattern of every element includes coupled power to neighbors which re-radiate and alter the effective excitation currents.

V Dipole over Ground Plane

Physical optics can be used to compute the pattern of a V dipole and used in Eq. (3-29) integral to compute tables of normalized mutual resistance. Below is an example of **VDIPMUT.** The reactance theorem is used with parallel wire dipoles using many segments to obtain an accurate value of reactance. In the example below 121 segments per pole were used for impedance and only 15 segments for pattern computations. Computing proper gain only requires correct values of resistance which can be obtained with few segments for impedance. The reactance computed may be incorrect but has no effect on the pattern gain. Smaller diameter elements require more segments in the impedance calculation for proper reactance values. The dipole couples to its image in the ground plane and it increases the input impedance as shown. VDPMUT computes mutual impedances directly without the integrals and produces inputs to GPDIRU more quickly.

```
Dipole length:          14.250 cm
Dipole Diameter:        0.5000
Center height:           8.000
Element tilt (deg):      20.00
Frequency (GHz):         1.000
Dipole Self impedance:        56.806    7.432
       Mutual impedance:      -9.589  -27.159
       Total impedance:       66.395   34.590
Spacing (wavel) Switch to vector effective height:   10.00
E-plane beamwidth:    76.99
H-plane beamwidth:   117.25
D-plane beamwidth:    95.29
Boresidght Gain (dB):   7.29
Number of Phi planes:   5

Direction of array:    0.00
Gain (@90) dB:   -8.22
Spacing (wavel.)  Resistance    Resist. dB
         0.500     3.8096E-01     -4.19
         0.600     2.2427E-01     -6.49
         0.700     1.0019E-01     -9.99
         0.800     1.5964E-02    -17.97
         0.900    -2.9191E-02    -15.35
         1.000    -4.2481E-02    -13.72
         1.100    -3.4818E-02    -14.58
         1.200    -1.7733E-02    -17.51
         1.300    -7.4279E-04    -31.29
         1.400     1.0266E-02    -19.89
         1.500     1.3415E-02    -18.72
         1.600     1.0101E-02    -19.96
         1.700     3.5479E-03    -24.50
         1.800    -2.7655E-03    -25.58
         1.900    -6.3742E-03    -21.96
         2.000    -6.4450E-03    -21.91
```

```
…

Direction of array:   90.00
Gain (@90) dB: -108.03
Spacing (wavel.)  Resistance    Resist. dB
         0.500     1.6568E-01     -7.81
         0.600    -1.0984E-02    -19.59
         0.700    -1.2509E-01     -9.03
         0.800    -1.7123E-01     -7.66
         0.900    -1.5843E-01     -8.00
         1.000    -1.0614E-01     -9.74
         1.100    -3.8363E-02    -14.16
         1.200     2.2703E-02    -16.44
         1.300     6.1630E-02    -12.10
         1.400     7.2380E-02    -11.40
         1.500     5.8078E-02    -12.36
         1.600     2.8421E-02    -15.46
         1.700    -4.2874E-03    -23.68
         1.800    -2.9180E-02    -15.35
         1.900    -3.9795E-02    -14.00
         2.000    -3.5281E-02    -14.52
         …
```

When we use the table output file from **VDIPMUT** in **GPDIRU,** we calculate the array gain efficiency. Using a 121 element (11 x 11) array of uniform amplitude elements, the directivity and efficiency are computed using Eq. (3-30).

Table 2 Directivity of Planar Array of V dipoles

```
XADEF file name: rect121.arr
Mutual Impedance File: vdipmut3.dat
Rotation of beam ellipse:    0.00
Element Directivity =    7.29
```

Frequency GHz	Directivity dB	Efficiency dB
1.00000	25.89	-2.23
1.10000	26.36	-1.76
1.20000	27.41	-0.70
1.30000	28.18	0.07
1.40000	28.66	0.54
1.50000	29.18	1.07
1.60000	29.72	1.60
1.70000	30.20	2.08
1.80000	30.52	2.40
1.90000	30.68	2.56
2.00000	29.40	1.28
2.10000	28.14	0.03
2.20000	27.79	-0.32
2.30000	27.73	-0.38
2.40000	27.78	-0.33
2.50000	27.90	-0.21
2.60000	28.04	-0.07
2.70000	28.18	0.06

```
   2.80000      28.02       -0.09
   2.90000      27.72       -0.40
   3.00000      27.58       -0.53
```

The column of frequency is the scaling constant on λ/2 element spacing. Elements spaced λ/2 in both the *x*- and *y*-axes has an effective gain of $4\pi(\lambda/2)(\lambda/2)/\lambda^2 = \pi$ or 5 dB. The array gain efficiency reduces the V dipole gain of 7.29 dB to approximately 5 dB.

Array of Microstrip Patches

Consider an array of microstrip patches on a dielectric substrate ER = 2.2 which has an element directivity of 7.5 dB for a square patch on an infinite ground plane. The patch width is 0.337λ and it would be reasonable to space elements λ/2. The directivity of a uniform amplitude square $0.25\lambda^2$ is π (or 5 dB) which means the efficiency for the array element needs to drop.

We can compute the radiation by a patch on an infinite ground plane by using the radiation of magnetic currents surrounding the edge of the patch (see Figure 6-2) scaled so that the radiated peak field equals gain (section 3.7, Diaz and Milligan, *Antenna Engineering using Physical Optics,* Artech 1996). By using PO the element pattern can be used with Eq. (3-29) of the two element array pattern integral to compute normalized mutual resistances (**SPIMUT** for a square patch, **RPIMUT** for a rectangular patch, and **CPIMUT** for a circular patch) versus spacing.

SPIMUT output (Square Patch, ER = 2.2)

```
 Operating Frequency (GHz):   1.000
 Effective patch width:      10.106 cm
 Spacing (wavel) Switch to vector effective height:    8.00
 E-plane beamwidth:    95.74
 H-plane beamwidth:    80.21
 D-plane beamwidth:    88.96
 Boresidght Gain (dB):   7.55
 Number of Phi planes:   5


 Direction of array:    0.00 (E-plane)
 Gain (@90) dB:    1.35
 Spacing (wavel.)  Resistance    Resist. dB
          0.500     1.0355E-01     -9.85
          0.550     1.3744E-02    -18.62
          0.600    -5.8152E-02    -12.35
          0.650    -1.1051E-01     -9.57
          0.700    -1.4300E-01     -8.45
          0.750    -1.5656E-01     -8.05
          0.800    -1.5321E-01     -8.15
          0.850    -1.3587E-01     -8.67
          0.900    -1.0808E-01     -9.66
          0.950    -7.3721E-02    -11.32
          1.000    -3.6701E-02    -14.35
          1.050    -6.6733E-04    -31.76
          1.100     3.1232E-02    -15.05
```

```
          1.150     5.6547E-02    -12.48
          1.200     7.3652E-02    -11.33
          1.250     8.1802E-02    -10.87
          1.300     8.1113E-02    -10.91
          1.350     7.2476E-02    -11.40
          1.400     5.7418E-02    -12.41
          1.450     3.7915E-02    -14.21
          1.500     1.6187E-02    -17.91
          1.550    -5.5197E-03    -22.58
          1.600    -2.5132E-02    -16.00
          1.650    -4.0923E-02    -13.88
          1.700    -5.1643E-02    -12.87
          1.750    -5.6604E-02    -12.47
          1.800    -5.5700E-02    -12.54
          1.850    -4.9384E-02    -13.06
          1.900    -3.8591E-02    -14.14
          1.950    -2.4620E-02    -16.09
          2.000    -8.9993E-03    -20.46

 Direction of array:   90.00 (H-plane)
 Gain (@90) dB:  -128.34
 Spacing (wavel.)  Resistance    Resist. dB
          0.500     2.8739E-01     -5.42
          0.550     1.9890E-01     -7.01
          0.600     1.1989E-01     -9.21
          0.650     5.2301E-02    -12.81
          0.700    -2.6334E-03    -25.79
          0.750    -4.4378E-02    -13.53
          0.800    -7.3084E-02    -11.36
          0.850    -8.9529E-02    -10.48
          0.900    -9.5021E-02    -10.22
          0.950    -9.1287E-02    -10.40
          1.000    -8.0329E-02    -10.95
          1.050    -6.4290E-02    -11.92
          1.100    -4.5311E-02    -13.44
          1.150    -2.5402E-02    -15.95
          1.200    -6.3337E-03    -21.98
          1.250     1.0448E-02    -19.81
          1.300     2.3880E-02    -16.22
          1.350     3.3304E-02    -14.78
          1.400     3.8468E-02    -14.15
          1.450     3.9494E-02    -14.03
          1.500     3.6823E-02    -14.34
          1.550     3.1144E-02    -15.07
          1.600     2.3313E-02    -16.32
          1.650     1.4266E-02    -18.46
          1.700     4.9371E-03    -23.07
          1.750    -3.8202E-03    -24.18
          1.800    -1.1291E-02    -19.47
          1.850    -1.6946E-02    -17.71
          1.900    -2.0467E-02    -16.89
          1.950    -2.1750E-02    -16.63
          2.000    -2.0899E-02    -16.80
```

Because the *H*-plane radiation at 90° is zero, it is impractical to use Eq. (1-51) to compute normalized mutual resistance and we have to settle on integration of the pattern Eq. (3-29).

A second method uses mutual coupling between patches using the reactance theorem (Section 2-4.4) of the equivalent magnetic currents to compute the normalized mutual impedance (**SPPMUT** square patch, **RPPMUT** rectangular patch, and **CPPMUT** circular patch) versus spacing that run more quickly.

SPPMUT output (Square patch ER = 2.2)

```
 Operating Frequency (GHz):   1.000
 Effective patch width:      10.106 cm
 Spacing (wavel) Switch to vector effective height:    8.00
 E-plane beamwidth:    95.74
 H-plane beamwidth:    80.21
 D-plane beamwidth:    88.96
 Boresidght Gain (dB):   7.55
 Number of Phi planes:   2


 Direction of array:    0.00
 Gain (@90) dB:    1.35
 Spacing (wavel.)  Resistance   Reactance  Resist. dB Spacing   Coupling
         0.5000   1.0755E-01  -3.2771E-01     -9.68    14.990    -15.06
         0.5500   1.4439E-02  -2.9701E-01    -18.40    16.489    -16.36
         0.6000  -6.2335E-02  -2.5433E-01    -12.05    17.988    -17.53
         0.6500  -1.2115E-01  -1.9998E-01     -9.17    19.487    -18.59
         0.7000  -1.5975E-01  -1.3738E-01     -7.97    20.985    -19.56
         0.7500  -1.7691E-01  -7.2079E-02     -7.52    22.484    -20.46
         0.8000  -1.7371E-01  -1.0429E-02     -7.60    23.983    -21.27
         0.8500  -1.5362E-01   4.2105E-02     -8.14    25.482    -22.02
         0.9000  -1.2149E-01   8.2015E-02     -9.15    26.981    -22.72
         0.9500  -8.2395E-02   1.0785E-01    -10.84    28.480    -23.36
         1.0000  -4.0880E-02   1.1971E-01    -13.88    29.979    -23.95
         1.0500  -7.4336E-04   1.1865E-01    -31.29    31.478    -24.50
         1.1000   3.4864E-02   1.0634E-01    -14.58    32.977    -25.02
         1.1500   6.3364E-02   8.4990E-02    -11.98    34.476    -25.51
         1.2000   8.2834E-02   5.7350E-02    -10.82    35.975    -25.96
         1.2500   9.2215E-02   2.6578E-02    -10.35    37.474    -26.39
         1.3000   9.1482E-02  -4.0418E-03    -10.39    38.973    -26.80
         1.3500   8.1647E-02  -3.1495E-02    -10.88    40.472    -27.19
         1.4000   6.4553E-02  -5.3394E-02    -11.90    41.971    -27.56
         1.4500   4.2544E-02  -6.8168E-02    -13.71    43.470    -27.91
         1.5000   1.8143E-02  -7.5082E-02    -17.41    44.969    -28.25
         1.5500  -6.1869E-03  -7.4157E-02    -22.09    46.468    -28.58
         1.6000  -2.8199E-02  -6.6067E-02    -15.50    47.967    -28.89
         1.6500  -4.5989E-02  -5.2047E-02    -13.37    49.466    -29.19
         1.7000  -5.8124E-02  -3.3797E-02    -12.36    50.965    -29.47
         1.7500  -6.3764E-02  -1.3339E-02    -11.95    52.464    -29.75
         1.8000  -6.2750E-02   7.1886E-03    -12.02    53.963    -30.02
         1.8500  -5.5597E-02   2.5762E-02    -12.55    55.462    -30.28
         1.9000  -4.3400E-02   4.0691E-02    -13.63    56.961    -30.53
```

```
        1.9500  -2.7662E-02   5.0767E-02    -15.58    58.460    -30.78
        2.0000  -1.0106E-02   5.5327E-02    -19.95    59.958    -31.01

 Direction of array:   90.00
 Gain (@90) dB: -128.34
 Spacing (wavel.)  Resistance   Reactance  Resist. dB Spacing   Coupling
        0.5000   3.2972E-01  -1.0342E-01     -4.82    14.990    -15.46
        0.5500   2.2173E-01  -1.7471E-01     -6.54    16.489    -17.05
        0.6000   1.3156E-01  -1.9951E-01     -8.81    17.988    -18.41
        0.6500   5.7175E-02  -1.9844E-01    -12.43    19.487    -19.64
        0.7000  -2.8914E-03  -1.8087E-01    -25.39    20.985    -20.80
        0.7500  -4.9110E-02  -1.5222E-01    -13.09    22.484    -21.90
        0.8000  -8.1558E-02  -1.1671E-01    -10.89    23.983    -22.94
        0.8500  -1.0056E-01  -7.8190E-02     -9.98    25.482    -23.93
        0.9000  -1.0712E-01  -4.0233E-02     -9.70    26.981    -24.87
        0.9500  -1.0301E-01  -5.8426E-03     -9.87    28.480    -25.77
        1.0000  -9.0560E-02   2.2779E-02    -10.43    29.979    -26.63
        1.0500  -7.2347E-02   4.4328E-02    -11.41    31.478    -27.45
        1.1000  -5.0901E-02   5.8320E-02    -12.93    32.977    -28.24
        1.1500  -2.8505E-02   6.4929E-02    -15.45    34.476    -29.00
        1.2000  -7.1060E-03   6.4825E-02    -21.48    35.975    -29.72
        1.2500   1.1729E-02   5.9037E-02    -19.31    37.474    -30.42
        1.3000   2.6833E-02   4.8849E-02    -15.71    38.973    -31.09
        1.3500   3.7461E-02   3.5707E-02    -14.26    40.472    -31.74
        1.4000   4.3302E-02   2.1113E-02    -13.63    41.971    -32.37
        1.4500   4.4471E-02   6.5093E-03    -13.52    43.470    -32.97
        1.5000   4.1461E-02  -6.8344E-03    -13.82    44.969    -33.56
        1.5500   3.5055E-02  -1.7910E-02    -14.55    46.468    -34.12
        1.6000   2.6230E-02  -2.6026E-02    -15.81    47.967    -34.67
        1.6500   1.6046E-02  -3.0826E-02    -17.95    49.466    -35.20
        1.7000   5.5525E-03  -3.2272E-02    -22.56    50.965    -35.72
        1.7500  -4.2964E-03  -3.0612E-02    -23.67    52.464    -36.22
        1.8000  -1.2702E-02  -2.6325E-02    -18.96    53.963    -36.70
        1.8500  -1.9069E-02  -2.0064E-02    -17.20    55.462    -37.18
        1.9000  -2.3035E-02  -1.2589E-02    -16.38    56.961    -37.64
        1.9500  -2.4483E-02  -4.6895E-03    -16.11    58.460    -38.09
        2.0000  -2.3525E-02   2.8802E-03    -16.28    59.958    -38.53
```

While the two methods do not produce exactly the same mutual resistance, they are close. We can also use **CBMUTU** $\cos^N(\theta/2)$ beam and **GBMUTU** Gaussian beam using the beamwidths to compute mutual resistances for the square patch. By using **GPDIRU** we compute the efficiency of a 121 element (11 x 11) rectangular array initially spaced λ/2. One array uses a uniform amplitude distribution and the second one a 25 dB Taylor distribution in both planes.

Table 3 121 Element Array of Linearly Polarized Square Patches on 2.2 Dielectric Constant Array Gain Efficiency versus Element Spacing: Integral of pattern, Mutual Coupling, Cosine beam pattern integral, Gaussian beam pattern integral dB

0.5λ	Integral	Integral	Coupling	Coupling	Cos(θ/2)	Cos(θ/2)	Gaussian	Gaussian
Spacing Ratio	Uniform eff. dB	Taylor 25 eff. dB	Uniform eff. dB	Taylor 25 eff. dB	Uniform eff. dB	Taylor 25 eff. dB	Uniform eff. dB	Taylor 25 eff. dB
1	-2.03	-2.07	-2.10	-2.18	-2.34	-2.42	-2.18	-2.24
1.1	-1.18	-1.24	-1.23	-1.29	-1.54	-1.6	-1.37	-1.42

1.2	-0.46	-0.51	-0.49	-0.51	-0.84	-0.86	-0.69	-0.7
1.3	0.13	0.15	0.25	0.24	-0.17	-0.18	-0.02	-0.01
1.4	1.04	0.93	0.88	0.93	0.4	0.45	0.53	0.6
1.5	1.3	1.41	1.56	1.63	0.98	1.04	1.1	1.18
1.6	2.46	2.33	2.11	2.29	1.45	1.58	1.55	1.72
1.7	2	2.38	2.68	2.97	1.9	2.1	1.99	2.23
1.8	3.39	3.44	3.11	3.61	2.24	2.57	2.3	2.68
1.9	1.8	1.62	2.66	2.29	2	1.78	1.87	1.55
2	-0.27	-0.35	-0.49	-0.46	-0.3	-0.29	-0.84	-0.82
2.1	-1.07	-1.12	-1	-1.04	-0.85	-0.9	-1.27	-1.33
2.2	0.01	-0.06	-0.22	-0.36	-0.34	-0.49	-0.5	-0.67
2.3	0.64	0.59	0.03	0.07	-0.22	-0.23	-0.3	-0.28
2.4	0.04	0.08	0.30	0.35	0.01	0	0.02	0.02
2.5	0.49	0.63	0.46	0.60	0.15	0.18	0.2	0.24
2.6	0.63	0.83	0.60	0.83	0.27	0.38	0.33	0.48
2.7	0.17	0.25	0.52	0.57	0.3	0.34	0.34	0.37
2.8	-0.61	-0.68	-1.08	-1.17	-0.4	-0.43	-0.57	-0.63
2.9	-1.45	-1.61	-1.83	-1.93	-0.82	-0.89	-1.04	-1.13
3	-1.25	-1.38	-1.05	-1.23	-0.64	-0.71	-0.75	-0.83
3.1	-0.4	-0.44	-0.59	-0.63	-0.31	-0.35	-0.29	-0.34
3.2	0.06	0.09	-0.27	-0.26	-0.15	-0.17	-0.09	-0.1
3.3	-0.14	-0.14	0.02	0.03	-0.06	-0.06	0.01	0.02
3.4	0.06	0.08	0.27	0.29	0.15	0.14	0.26	0.26
3.5	0.63	0.7	0.51	0.54	0.31	0.32	0.45	0.47
3.6	0.94	1.01	0.71	0.77	0.38	0.42	0.5	0.55
3.7	0.82	0.84	0.93	1.00	0.52	0.57	0.65	0.71
3.8	0.57	0.56	1.09	1.20	0.8	0.83	0.99	1.02
3.9	0.87	0.92	1.06	1.00	0.59	0.6	0.65	0.65
4	0.97	1.09	0.18	0.24	0.1	0.11	0.01	0

TICRA cut file mutual resistance

GRASP contains a number of sources that can be used to generate an output *.cut file. Cut files scan θ and step ϕ to generate a full radiation sphere pattern. For an example of using a *.cut file, we use a 6-turn helix with a 14° wire pitch angle and a one wavelength circumference to produce the pattern.

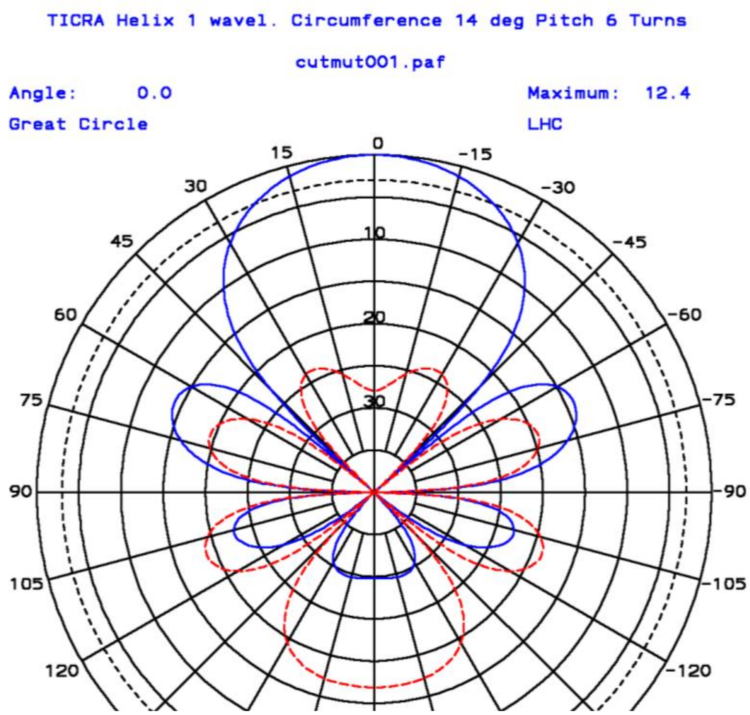

Figure 13 GRASP Helix Feed

The pattern has a null at 90° which means we cannot use the approximate vector effective length formula (Eq. (1-51)) to compute widely spaced normalized mutual resistance. The pattern of a two-element array needs to be calculated Eq. (3-29) using the tabulated pattern (GRASP cut file). GRASP was used to compute a complete spherical pattern of the helix and the results were entered into the program **CUTMUT** to generate a table of mutual resistance versus spacing file for use in **GPDIRU.**

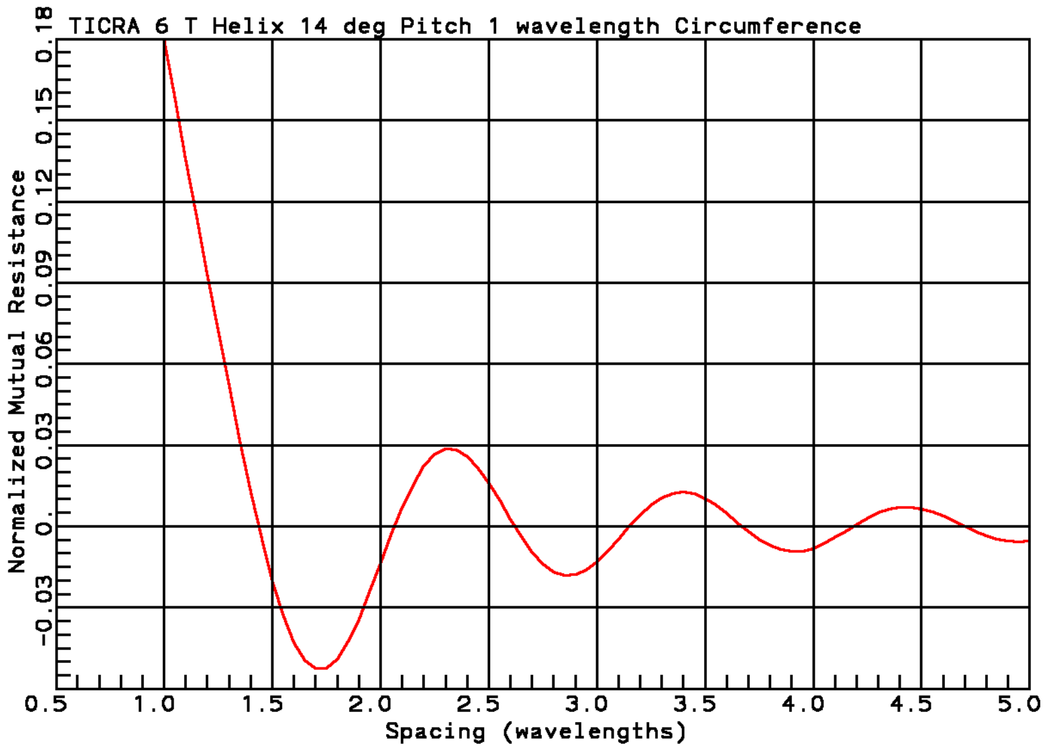

Figure 14 Normalized mutual resistance of two element array of GRASP 6 turn helix feed pattern

Figure 14 illustrates that the normalized mutual resistance rises rapidly as the elements are brought close together. Array gain efficiency falls rapidly as the helices are crowded together in an 11 x 11 uniform amplitude rectangular array.

Table 4 Directivity of Planar Array of GRASP 6 turn Helix of 121 elements in a Uniform Amplitude Array

```
 XADEF file name: recth121.arr
 Mutual Impedance File: cutmuth6.dat
 Rotation of beam ellipse:    0.00
 Element Directivity =   12.42
```

Frequency GHz	Directivity dB	Efficiency dB
0.70000	28.72	-4.53
0.75000	29.29	-3.96
0.80000	29.82	-3.43
0.85000	30.30	-2.95
0.90000	30.74	-2.51
0.95000	31.16	-2.09
1.00000	30.96	-2.29
1.05000	30.75	-2.50
1.10000	31.04	-2.21
1.15000	31.75	-1.50

1.20000	32.48	-0.77
1.25000	33.05	-0.20
1.30000	33.53	0.28
1.35000	33.89	0.64
1.40000	33.65	0.40
1.45000	33.34	0.09
1.50000	33.14	-0.11
1.55000	33.17	-0.08
1.60000	33.37	0.11
1.65000	33.66	0.41
1.70000	33.80	0.54
1.75000	33.86	0.61
1.80000	33.97	0.72
1.85000	34.08	0.82
1.90000	34.07	0.82
1.95000	33.85	0.60
2.00000	33.65	0.40
2.05000	33.48	0.23
2.10000	33.33	0.08
2.15000	33.20	-0.05
2.20000	33.03	-0.22
2.25000	32.82	-0.43
2.30000	32.71	-0.54
2.35000	32.63	-0.62
2.40000	32.66	-0.59
2.45000	32.77	-0.48
2.50000	32.90	-0.35

Horn Array Elements

The aperture area of a horn at a given gain is greater than the uniform amplitude projected area because it has amplitude taper which reduces its gain. This means when we use horns in an array, we expect the array efficiency loss to be near zero and the array gain equal to the number of elements times the element gain. Corrugated horns have very low coupling between elements in a planar array because the gain at 90° is low. We evaluate the normalized mutual resistance by either integrating the pattern of a two-element array of equal elements by using Eq. (3-29) or compute the coupling between horns by using the reactance theorem analysis of coupling between equivalent currents in the receive aperture and radiation by the currents in the transmit aperture. In both cases the currents are computing using the methods from Diaz and Milligan, *Antenna Engineering using Physical Optics,* Artech House, 1996.

13 dB Gain Horns

The program **CHPMUT** computed the coupling between two 13 dB gain corrugated horns from which the normalized mutual impedance was found. The gain at 90° is -22.8 dB which means the coupling even when the edges (aperture plus corrugation depth) touch the coupling is less than 60 dB (see table below).

```
Circular Corrugated horn CHPMUT
Diameter:                 50.0000 cm
Slant Radius:             184.000 cm
Depth of corrugations:      7.4900
X axis Ampl.(dB):    0.00 Phase:    0.00
Y axis Ampl.(dB): -100.00 Phase:    0.00 Rotation:    0.00
Cutoff frequency (GHz):     0.4502
Wall reactance:           373491.7
Hybrid Ratio:               0.9994

Operating Frequency (GHz):   1.000
Edge length of patches:     5.0000
Boresight gain (dB):   13.00
Diameter (wavelengths):     1.67

Direction of array:    0.00
Gain (@90) dB:  -22.78
Spacing (wavel.)  Resistance   Reactance  Resist. dB Spacing   Coupling
        1.7000  -5.9286E-04   1.4214E-03    -32.27    50.965    -62.27
        1.8000   5.6669E-04   1.2604E-03    -32.47    53.963    -63.21
        1.9000   1.1348E-03   4.7452E-04    -29.45    56.961    -64.22
        2.0000   1.0459E-03  -3.4984E-04    -29.81    59.958    -65.17
        2.1000   5.2037E-04  -8.5031E-04    -32.84    62.956    -66.05
        2.2000  -1.2332E-04  -8.9958E-04    -39.09    65.954    -66.86
        2.3000  -6.0423E-04  -5.7282E-04    -32.19    68.952    -67.61
        2.4000  -7.6488E-04  -6.9063E-05    -31.16    71.950    -68.31
        2.5000  -5.9835E-04   3.8613E-04    -32.23    74.948    -68.97
        2.6000  -2.2061E-04   6.2562E-04    -36.56    77.946    -69.59
        2.7000   1.9292E-04   5.8978E-04    -37.15    80.944    -70.17
        2.8000   4.8001E-04   3.3015E-04    -33.19    83.942    -70.71
        2.9000   5.4818E-04  -2.5910E-05    -32.61    86.940    -71.23
        3.0000   3.9829E-04  -3.3198E-04    -34.00    89.938    -71.73
```

The program **CHIMUT** computes the normalized mutual resistance by using Eq. (3-29) for a two-element array using identical corrugated horns patterns evaluated using the PO currents of the aperture. Because the normalized mutual resistance is small, numerical integration is increasingly challenging as the horns are increasing separated.

```
Circular Corrugated horn CHIMUT
Diameter:                 50.0000 cm
Slant Radius:             184.000 cm
Depth of corrugations:      7.4900
X axis Ampl.(dB):    0.00 Phase:    0.00
Y axis Ampl.(dB): -100.00 Phase:    0.00 Rotation:    0.00
Cutoff frequency (GHz):     0.4502
Wall reactance:           373491.7
Hybrid Ratio:               0.9994

Operating Frequency (GHz):   1.000
Edge length of patches:     5.0000
Boresight gain (dB):   13.00
Diameter (wavelengths):     1.67
```

```
Direction of array:    0.00
Gain (@90) dB:  -22.78
Spacing (wavel.)  Resistance    Resist. dB
         1.700    -6.5735E-04    -31.82
         1.800    -5.0472E-04    -32.97
         1.900    -1.2642E-04    -38.98
         2.000     1.6772E-04    -37.75
         2.100     2.9721E-04    -35.27
         2.200     2.8811E-04    -35.40
         2.300     1.9647E-04    -37.07
         2.400     7.2847E-05    -41.38
         2.500    -4.6235E-05    -43.35
         2.600    -1.3557E-04    -38.68
         2.700    -1.7813E-04    -37.49
         2.800    -1.6757E-04    -37.76
         2.900    -1.0421E-04    -39.82
         3.000    -1.0900E-05    -49.63
```

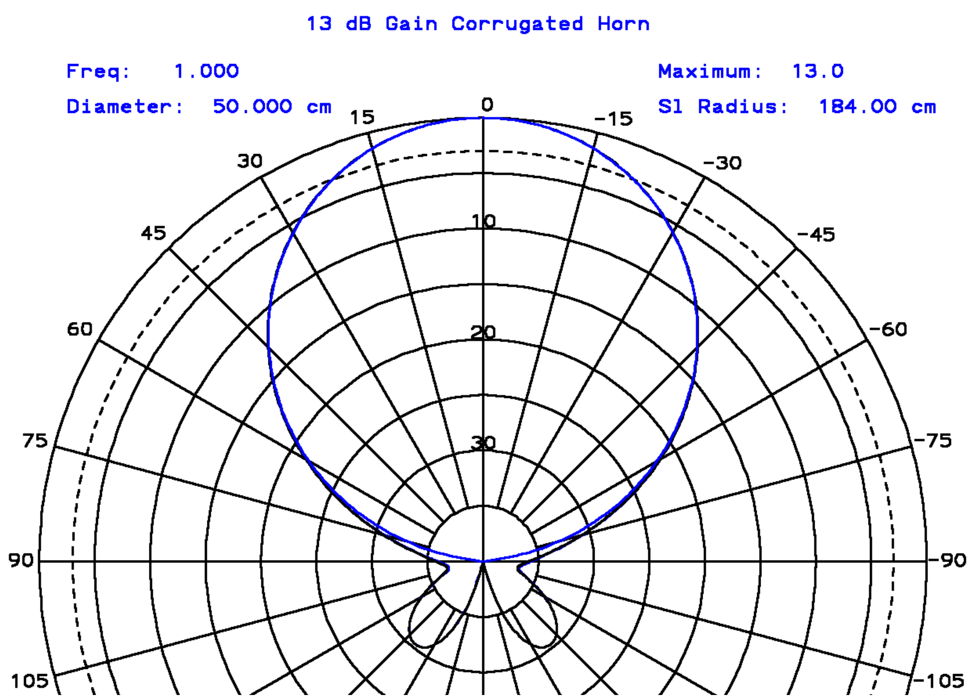

Figure 15 13 dB Corrugated Horn Pattern (black) compared to Gaussian Beam (blue)

Circular Waveguide Horn TE_{11} Mode

```
Diameter:                    46.7600 cm
Slant Radius:                182.500 cm
Bessel order, Zero:            1   1
Smooth wall TE mode
X axis Ampl.(dB):    0.00 Phase:    0.00
Y axis Ampl.(dB): -100.00 Phase:    0.00 Rotation:    0.00

Operating Frequency (GHz):   1.000
```

```
Edge length of patches:     4.0000
Boresight gain (dB):   13.03
Diameter (wavelengths):     1.56

Direction of array:    0.00
Gain (@90) dB:  -10.20
Spacing (wavel.)  Resistance   Reactance  Resist. dB Spacing   Coupling
        1.6000  -1.9042E-03  -1.8584E-02    -27.20    47.967    -40.59
        1.7000  -9.8674E-03  -1.1018E-02    -20.06    50.965    -42.62
        1.8000  -1.2376E-02  -2.8158E-03    -19.07    53.963    -43.95
        1.9000  -1.0429E-02   4.3087E-03    -19.82    56.961    -44.97
        2.0000  -5.4653E-03   8.6578E-03    -22.62    59.958    -45.82
        2.1000   5.0497E-04   9.4023E-03    -32.97    62.956    -46.54
        2.2000   5.4384E-03   6.8456E-03    -22.65    65.954    -47.19
        2.3000   7.8568E-03   2.2664E-03    -21.05    68.952    -47.77
        2.4000   7.2550E-03  -2.5549E-03    -21.39    71.950    -48.30
        2.5000   4.1762E-03  -5.9493E-03    -23.79    74.948    -48.79
        2.6000  -6.0311E-05  -6.8955E-03    -42.20    77.946    -49.25
        2.7000  -3.8676E-03  -5.3031E-03    -24.13    80.944    -49.68
        2.8000  -5.9496E-03  -1.9635E-03    -22.26    83.942    -50.08
        2.9000  -5.7210E-03   1.7925E-03    -22.43    86.940    -50.46
        3.0000  -3.4537E-03   4.5965E-03    -24.62    89.938    -50.83

Direction of array:   90.00
Gain (@90) dB:  -18.40
Spacing (wavel.)  Resistance   Reactance  Resist. dB Spacing   Coupling
        1.6000   1.7412E-03  -2.0678E-03    -27.59    47.967    -57.38
        1.7000  -3.9773E-04  -2.1300E-03    -34.00    50.965    -59.30
        1.8000  -1.5199E-03  -9.6568E-04    -28.18    53.963    -60.91
        1.9000  -1.5042E-03   3.3807E-04    -28.23    56.961    -62.26
        2.0000  -7.4616E-04   1.1278E-03    -31.27    59.958    -63.40
        2.1000   1.9752E-04   1.1935E-03    -37.04    62.956    -64.37
        2.2000   8.6059E-04   6.8462E-04    -30.65    65.954    -65.20
        2.3000   1.0114E-03  -5.1641E-05    -29.95    68.952    -65.91
        2.4000   6.8077E-04  -6.5196E-04    -31.67    71.950    -66.53
        2.5000   9.1186E-05  -8.8012E-04    -40.40    74.948    -67.08
        2.6000  -4.6716E-04  -6.9375E-04    -33.31    77.946    -67.57
        2.7000  -7.6204E-04  -2.2674E-04    -31.18    80.944    -68.01
        2.8000  -7.0119E-04   2.9123E-04    -31.54    83.942    -68.41
        2.9000  -3.4872E-04   6.3890E-04    -34.58    86.940    -68.78
        3.0000   1.2062E-04   6.8952E-04    -39.19    89.938    -69.12
```

The smooth wall 13 dB circular horn has higher lobes at 90° in the *E*-plane than the *H*-plane, Figure 16 which increases coupling and the normalized mutual resistance when arrayed in the *E*-plane. Table 5 illustrates higher ripple in array efficiency than the corrugated horn due to the higher coupling. However, the ripple is insignificant. A rectangular 13-dB horn designed for approximately equal beamwidths has higher coupling at 90° than a 13-dB circular horn. The efficiency ripple of the rectangular horn versus element spacing, Table 5, is greater than the small gain TE_{11} mode circular horn.

While the array efficiency ripple of an array of helical wire antennas is high when they are closely spaced, when we separate them by an area equal to a similar gain corrugated horn it drops almost to the same level.

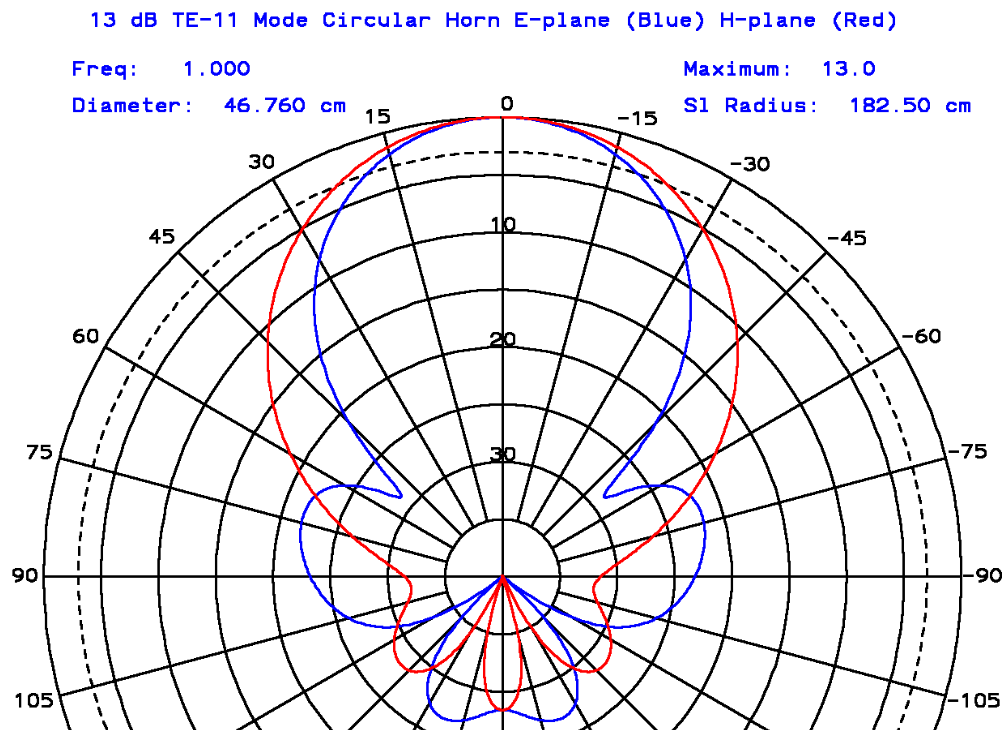

Figure 16 13 dB TE_{11} Mode Circular Horn

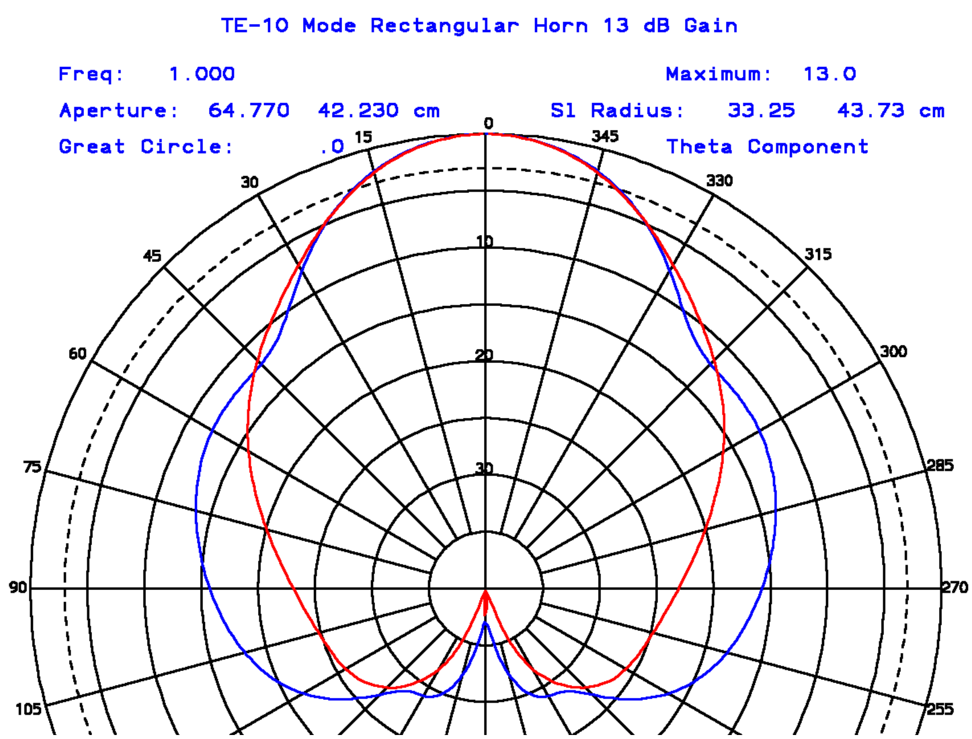

Figure 17 13 dB TE_{10} Mode Rectangular Horn

Table 5 13 dB Gain Horn Elements in 7 x 7 Rectangular Uniform Amplitude Array Gain Efficiency versus Element Spacing: 2.17λ spacing for Corrugated Horn (1.5λ x 2.3λ Rectangular Horn Spacing) (12.4 dB Helix)

2.17λ Sq. Spacing Ratio	Gaussian 13 dB Gain	Integral Corrugated Horn	Coupling Corrugated Horn	Coupling TE_{11} Cir. Horn	Integral Rectangular Horn	Coupling Rectangular Horn	Integral GRASP Helix
1	0	0	-0.01	0.09	-0.04	0.17	-0.11
1.1	0	0	0.02	-0.17	0.36	0.46	-0.65
1.2	0	0	0	0.03	0.47	0.24	0.05
1.3	0	0	-0.01	0.09	0.19	-0.02	0.41
1.4	0	0	0	0.03	-0.6	-0.52	0.14
1.5	0	0	0.01	-0.15	-0.08	0.08	-0.28
1.6	0	0	0	0.04	-0.12	-0.08	-0.06
1.7	0	0	0	-0.07	0.15	0.08	-0.04
1.8	0	0	-0.01	0.08	0.2	0.16	0.13
1.9	0	0	0	0.01	0.27	0.27	0.1
2	0	0	0	-0.03	-0.06	-0.19	-0.1
2.1	0	0	0.01	-0.07	-0.22	-0.23	-0.12
2.2	0	0	0	0.05	-0.2	-0.09	0.03
2.3	0	0	0	0.08	0.08	0.17	0.18
2.4	0	0	0	-0.08	0.03	0.03	-0.08
2.5	0	0	0	-0.03	0.19	0.1	-0.07
2.6	0	0	0	0.03	0.26	0.14	-0.01
2.7	0	0	0	0.03	-0.22	-0.17	0.06
2.8	0	0	0	0	-0.19	-0.14	0.02
2.9	0	0	0	-0.01	-0.02	0	0
3	0	0	0	-0.03	0.01	0.02	-0.05

When the horn gain increases to 16 dB, array efficiency ripple decreases to insignificant values. Increasing the horn gains even higher 16 dB gain horns allows the continued use of (element gain)*(Number of elements) for the gain of an array.

Table 6 16 dB Gain Horn Elements in 7 x 7 Rectangular Uniform Amplitude Array Gain Efficiency versus Element Spacing: 2.9λ spacing for Corrugated Horn 2.3λ spacing for TE_{11} Circular Horn, and 2λ x 3λ for Rectangular Horn Spacing

Spacing Ratio	Integral Corrugated Horn	Coupling Corrugated Horn	Integral TE_{11} Cir. Horn	Coupling TE_{11} Cir. Horn	Integral Rectangular Horn	Coupling Rectangular Horn
1	0	0	-0.02	0.07	-0.02	0.17
1.1	-0.01	0	0.01	0.01	0.05	0.06
1.2	0	0	0.01	-0.03	0	-0.01
1.3	0	0	0	-0.01	-0.01	-0.03
1.4	0	0	-0.01	0.03	-0.02	-0.05
1.5	0	0	0.01	0	0.02	0.05

1.6	0	0	-0.01	0.01	0.04	0.03
1.7	0	0	0.01	-0.02	-0.02	-0.01
1.8	0	0	-0.01	0.01	-0.02	-0.03
1.9	0	0	0	0	0.01	-0.02
2	0	0	0	0	0	0.04
2.1	0	0	0.01	-0.02	-0.01	0.01
2.2	0	0	-0.01	0.01	0	-0.01
2.3	0	0	0	0	0	-0.01
2.4	0	0	0	0	0	-0.01
2.5	0	0	0	-0.01	0.01	0.01
2.6	0	0	0	0	0	0.01
2.7	0	0	0	0	0	0
2.8	0	0	0	0.01	0	-0.02
2.9	0	0	0	0	0.01	-0.01
3	0	0	0.01	-0.01	0.01	0.02

Element Gain variation due to unbalanced Coupling

Elements in the corners and edges have fewer elements around them to couple to and dissipate less power in surrounding elements. Because power is dissipated in surrounding elements, the level is not affected by the feeding coefficients. All feeding coefficients in Eq. (3-30) are one, the numerator and first denominator summations are eliminated as each element is considered one at a time.

Executables to Compute Individual Array Element Amplitudes due to Coupling

GPDIRE: Using equal beamwidth elements and normalized mutual resistances data

GPDIRUE: Using unequal beamwidth elements and normalized mutual resistances data

Table 7 Element Gains due to Mutual Resistances for 90° beamwidth elements

```
Directivity of Planar Array

 XADEF file name: sq225n.arr
 Mutual Impedance File: gbmutn90.dat
 Element Directivity =    7.09

  No.  Element Location cm      Gain dB  Eff (dB)
  113  0.00000E+00  0.00000E+00   8.451    1.365
  114  1.50000E+01  0.00000E+00   8.516    1.430
  115  3.00000E+01  0.00000E+00   8.507    1.422
  116  4.50000E+01  0.00000E+00   8.401    1.315
  117  6.00000E+01  0.00000E+00   8.499    1.414
  118  7.50000E+01  0.00000E+00   8.582    1.496
  119  9.00000E+01  0.00000E+00   8.530    1.444
```

120	1.05000E+02	0.00000E+00	8.029	0.944
128	0.00000E+00	1.50000E+01	8.516	1.430
129	1.50000E+01	1.50000E+01	8.533	1.448
130	3.00000E+01	1.50000E+01	8.539	1.453
131	4.50000E+01	1.50000E+01	8.481	1.396
132	6.00000E+01	1.50000E+01	8.455	1.370
133	7.50000E+01	1.50000E+01	8.675	1.590
134	9.00000E+01	1.50000E+01	8.654	1.568
135	1.05000E+02	1.50000E+01	8.052	0.966
143	0.00000E+00	3.00000E+01	8.507	1.422
144	1.50000E+01	3.00000E+01	8.539	1.453
145	3.00000E+01	3.00000E+01	8.562	1.476
146	4.50000E+01	3.00000E+01	8.453	1.368
147	6.00000E+01	3.00000E+01	8.469	1.383
148	7.50000E+01	3.00000E+01	8.695	1.610
149	9.00000E+01	3.00000E+01	8.710	1.624
150	1.05000E+02	3.00000E+01	8.083	0.997
158	0.00000E+00	4.50000E+01	8.401	1.315
159	1.50000E+01	4.50000E+01	8.481	1.396
160	3.00000E+01	4.50000E+01	8.453	1.368
161	4.50000E+01	4.50000E+01	8.360	1.275
162	6.00000E+01	4.50000E+01	8.451	1.365
163	7.50000E+01	4.50000E+01	8.564	1.478
164	9.00000E+01	4.50000E+01	8.520	1.434
165	1.05000E+02	4.50000E+01	8.023	0.937
173	0.00000E+00	6.00000E+01	8.499	1.414
174	1.50000E+01	6.00000E+01	8.455	1.370
175	3.00000E+01	6.00000E+01	8.469	1.383
176	4.50000E+01	6.00000E+01	8.451	1.365
177	6.00000E+01	6.00000E+01	8.412	1.326
178	7.50000E+01	6.00000E+01	8.582	1.496
179	9.00000E+01	6.00000E+01	8.531	1.446
180	1.05000E+02	6.00000E+01	8.001	0.915
188	0.00000E+00	7.50000E+01	8.582	1.496
189	1.50000E+01	7.50000E+01	8.675	1.590
190	3.00000E+01	7.50000E+01	8.695	1.610
191	4.50000E+01	7.50000E+01	8.564	1.478
192	6.00000E+01	7.50000E+01	8.582	1.496
193	7.50000E+01	7.50000E+01	8.770	1.685
194	9.00000E+01	7.50000E+01	8.838	1.752
195	1.05000E+02	7.50000E+01	8.143	1.058
203	0.00000E+00	9.00000E+01	8.530	1.444
204	1.50000E+01	9.00000E+01	8.654	1.568
205	3.00000E+01	9.00000E+01	8.710	1.624
206	4.50000E+01	9.00000E+01	8.520	1.434
207	6.00000E+01	9.00000E+01	8.531	1.446
208	7.50000E+01	9.00000E+01	8.838	1.752
209	9.00000E+01	9.00000E+01	8.795	1.709
210	1.05000E+02	9.00000E+01	8.171	1.085
218	0.00000E+00	1.05000E+02	8.029	0.944
219	1.50000E+01	1.05000E+02	8.052	0.966
220	3.00000E+01	1.05000E+02	8.083	0.997
221	4.50000E+01	1.05000E+02	8.023	0.937
222	6.00000E+01	1.05000E+02	8.001	0.915

```
  223  7.50000E+01  1.05000E+02   8.143    1.058
  224  9.00000E+01  1.05000E+02   8.171    1.085
  225  1.05000E+02  1.05000E+02   7.705    0.620
 Average element gain:    8.435 1 Sigma:   0.252
 Average efficiency dB:   1.350

 Frequency:    1.500 Directivity:    30.68 Efficiency:    1.41
```

Table 7 shows the effective element gains due to the combination of an element and its neighbors at a spacing which produces increased gain relative to (number of elements)*(element gain of isolated antenna). Because the array is symmetrical only one-fourth of the elements are listed. The directivity efficiency is 1.35 dB for a uniform distribution and 1.41 dB for the dual 30 dB Taylor distributions. The coupling causes an equivalent 0.25 dB random amplitude variation, although it systematic and not random.