1-8.1.2 Aperture Approximation of Directivity given Beamwidths

By using the approximation *u* = sin *u* for small angles in the uniform aperture distribution, the half-power beamwidth can be estimated (see Section 2.2 or Section 4.2) as

$$\mathrm{HPBW} = 50.76° \frac{\lambda}{a}$$

Note that we have ignored the $(1+\cos\theta)/2$ pattern of the Huygens source, which reduces the beamwidth for radiation from small apertures.

We can use the result above to estimate the directivity of a rectangular aperture from beamwidths. The directivity of a uniform distribution is

$$Directivity = \frac{4\pi ab}{\lambda^2}$$

We solve for the aperture dimension divided by wavelength.

$$\frac{a}{\lambda} = \frac{50.76^\circ}{HPBW}$$

By substituting in the directivity formula and designating the beamwidths by θ_1 and θ_2 in the principal planes, we obtain a formula for directivity given beamwidths.

$$Directivity = \frac{32,375}{\theta_1\theta_2}$$

Although it has been derived for an aperture with a uniform distribution, it can be used with other distributions for an approximation. This differs from the Kraus estimate found by considering all power to be within the 3-dB beamwidth, [Eq. (1-19)] by 1 dB.

The beamwidth equation for a uniform circular distribution of diameter *D* (see Section 4.16) is

$$\frac{D}{\lambda} = \frac{58.95}{HPBW}$$

The uniform circular aperture has the directivity.

$$Directivity = \left(\frac{\pi D}{\lambda}\right)^2$$

When we gather terms, we compute directivity by using the beamwidths θ_1 and θ_2 in the principal planes.

$$Directivity = \frac{34,300}{\theta_1\theta_2}$$

This directivity differs by 0.25 dB from the one derived from a rectangular aperture. If we know the aperture distribution, we will use aperture efficiencies to determine directivity.

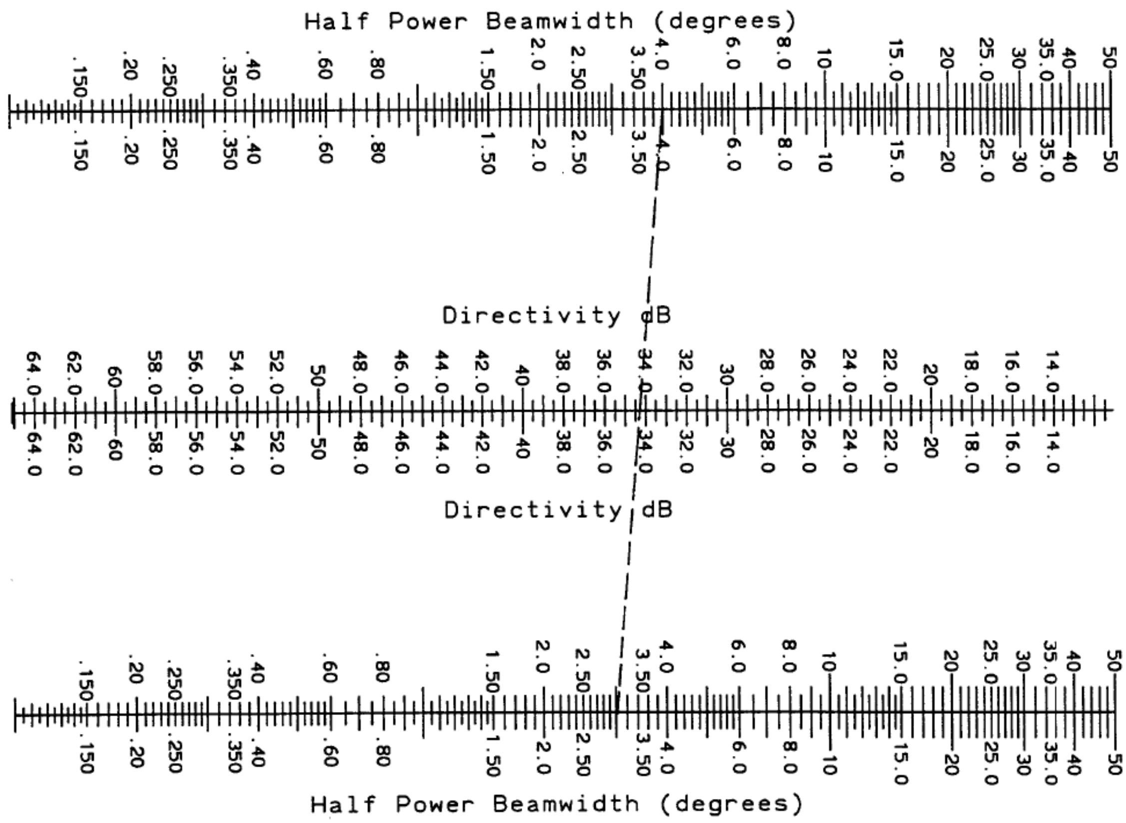

Figure 1-2a Directivity given *E*- and *H*-plane beamwidths